

MATH 100
Final Examination
December 6, 2013

Instructions:

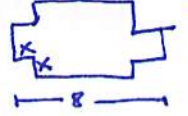
- ◆ You have **three** (3) hours to complete the examination.
- ◆ You may not leave during the final fifteen (15) minutes of the examination.
- ◆ All questions are of equal value, but not of equal difficulty. Choose carefully, and use your time wisely.
- ◆ A box **must** be placed around your final answer to each question.
- ◆ Part marks can only be given if your rough work is shown. Full marks will **not** be given for unsupported answers.
- ◆ If you are unsure of anything, **ask!**

- 1) Find $\lim_{\theta \rightarrow \infty} \sqrt{3-\theta}$ $\sqrt{-\infty} \therefore$ DNE or graphically  DNE out here $\rightarrow$
- 2) Find $\lim_{t \rightarrow 0} \frac{\sin t}{t^2 + 3t} = \lim_{t \rightarrow 0} \frac{\sin t}{t} \cdot \frac{1}{t+3} = \boxed{\frac{1}{3}}$
- 3) Find $f'(x)$ if $f(x) = \frac{3x^2 \cos x}{2x-1}$. Do not simplify your result.

$$f'(x) = \frac{(2x-1)[3x^2 - \sin x + 6\cos x \cdot 6x] - 3x^2 \cos x \cdot 2}{(2x-1)^2}$$
- 4) Find $\int \frac{1}{x^3} - \cos 4x \, dx = \boxed{-\frac{1}{2x^2} - \frac{\sin 4x}{4} + C}$
- 5) What is the average rate of change of $f(x) = 10\sqrt{x}$ on the interval $[0, 4]$? What is the instantaneous rate of change of $f(x)$ at 4? $\text{avg} \Rightarrow \frac{\Delta y}{\Delta x} = \boxed{5}$ $\text{inst} \Rightarrow \frac{dy}{dx} = \boxed{\frac{5}{2}}$
- 6) Find and classify the discontinuities (if any) in $f(x) = \frac{x}{2x^2+x}$
 $2x^2+x \cong 0 \Rightarrow x(2x+1) \cong 0 \Rightarrow \begin{cases} x=0 \Rightarrow \text{hole} \\ x=-1/2 \Rightarrow \text{asymptote} \end{cases}$
- 7) Find $f'(\theta)$ if $f(\theta) = \tan^2(\cos 3\theta + \sqrt{\theta})$. Do not simplify your result.

$$f'(\theta) = 2 \tan(\cos 3\theta + \sqrt{\theta}) \cdot \sec^2(\cos 3\theta + \sqrt{\theta}) \cdot (-\sin 3\theta \cdot 3 + \frac{1}{2\sqrt{\theta}})$$
- 8) Find $\int \frac{\sin(\frac{5}{x})}{x^2} \, dx = \boxed{+\frac{1}{5} \cos(\frac{5}{x}) + C}$
- 9) Find $\lim_{x \rightarrow -\infty} \frac{\sqrt{5x^2-2}}{x+3} = \boxed{-\sqrt{5}}$
- 10) Find $\frac{dy}{dx}$ if $y - x^2 = \sin xy$. $y' - 2x = \cos xy [x y' + y]$
 $y' - x \cos xy \cdot y' = y \cos xy + 2x$

$$y' = \frac{y \cos xy + 2x}{1 - x \cos xy}$$

12) 
$$V = (8-2x)(3-2x)x$$
$$= 4x^3 - 22x^2 + 24x$$
$$V' = 12x^2 - 44x + 24$$
$$= 3x^2 - 11x + 6 \geq 0$$
$$(3x-2)(x-3) = 0$$
$$x = 2/3 \quad x = 3$$
$$V = 7.907 \text{ Not possible}$$

@ endpoints $x=0$ & $x=1.5$, $V=0$

11) Draw a diagram illustrating the equation $f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$. Label each of the quantities in the equation on the diagram.

12) An open box is to be made from a 3 ft by 8 ft rectangular sheet of metal by cutting out squares of equal size from the four corners and then bending up the sides. What is the largest volume the box could contain?

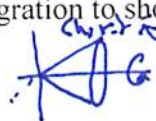
13) Derive **either** the formula,

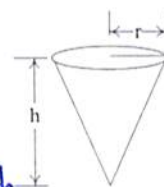
$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$

or the formula,

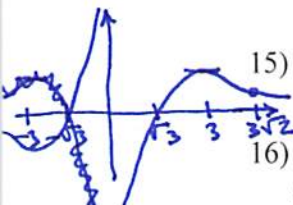
$$L = \int \sqrt{1 + [f'(x)]^2} dx$$

14) Use a suitable integration to show that the volume of a cone of height h and radius r is $\frac{\pi r^2 h}{3}$.


$$y = \frac{r}{h}x \Rightarrow V = \pi \int_0^h \left(\frac{r}{h}x\right)^2 dx$$
$$= \pi \frac{r^2}{h^2} \left[\frac{x^3}{3}\right]_0^h = \frac{\pi r^2}{h^2} \frac{h^3}{3} = \frac{\pi r^2 h}{3}$$



15) Sketch the graph of $y = \frac{x^2 - 3}{x^3}$. Label all points of interest.



16) A spherical balloon is being inflated by pumping in air at a rate of 2 cubic cm per second. At what rate is its perimeter increasing when the radius is 3 cm?

16) $V = \frac{4}{3}\pi r^3$
$$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$$
$$2 = 2\pi r^2 \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{1}{\pi r^2}$$
$$= \frac{1}{\pi (3)^2} = \frac{1}{9\pi} \text{ cm/s}$$
$$\approx 0.035 \text{ cm/s}$$

17) Find the area enclosed by the curves $y = 2 + |x-1|$ and $y = -\frac{1}{5}x + 7$.

$$A = \int_{-5}^1 \left(-\frac{x}{5} + 7\right) - (3-x) dx + \int_1^5 \left(-\frac{x}{5} + 7\right) - (4+x) dx$$

18) In strip mining the cost is proportional to the depth of the mine. Suppose that at a particular site the value of the ore increases as the square root of the depth and that at a depth of 25 feet the cost of mining is \$1.25/cubic unit, and the value is \$5.00/cubic unit.

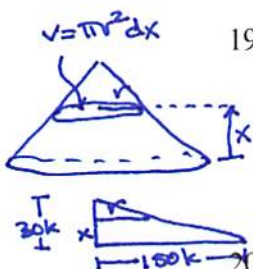
$$\text{Profit} = V - C = \sqrt{h} - 0.05h$$

a) At what depth will the mine cease to be profitable? $P=0 = \sqrt{h} - 0.05h \Rightarrow h = 400 \text{ ft}$

b) At what depth will the greatest profit be made, and what will that profit be? $\frac{dP}{dh} \geq 0 \Rightarrow h = 100$

c) If mining stops as soon as it is unprofitable what will be the total profit over the life of the mine? $P_{\text{tot}} = \int_0^{400} (\sqrt{h} - 0.05h) dh = \left[\frac{2}{3}h^{3/2} - 0.025h^2\right]_0^{400} \approx 1333 \$$

19) Geologists, when considering the origin of mountain ranges, estimate the energy required to produce a mountain. If we model Mt. Everest as a right circular cone of height 30,000 feet and radius 150,000 feet with a uniform density of 200 pounds per cubic foot, how much work was required to lift the material in Mt. Everest to its present position if it was all initially at sea level?



$$W = \int_0^{30k} 5^2 \pi \int_0^x (50k)^2 x - 60kx^2 + x^3 dx$$

$$\approx 1.6875 \times 10^{18} \pi \text{ ft-lbs}$$

20) Sketch the graph of $x^{2/3} + y^{2/3} = 4$. Label all points of interest.

