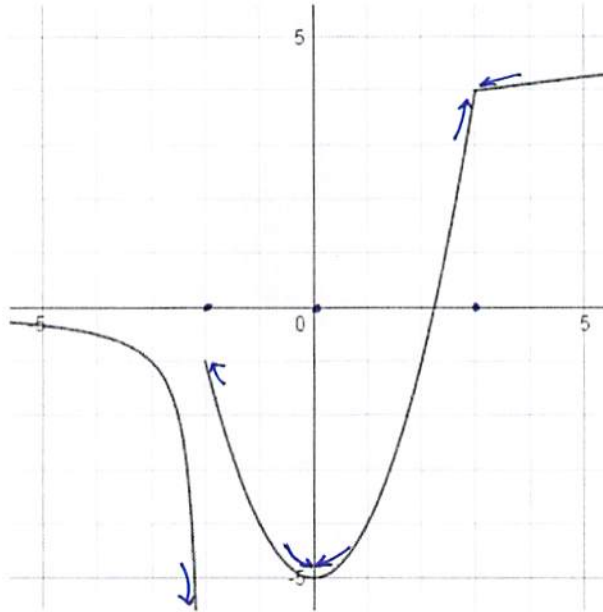


Homework set 2: Limits

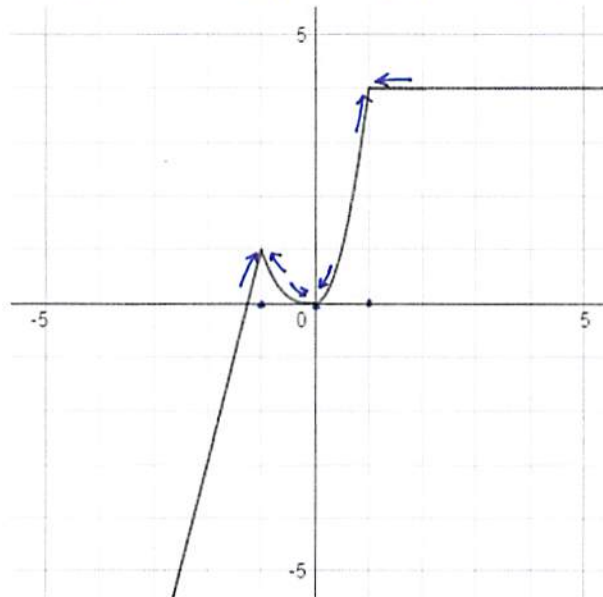
1. Find each of the following limits for the function $f(x)$ shown below:

$$\begin{array}{lllllll} \lim_{x \rightarrow 0^-} f(x), & \lim_{x \rightarrow -2} f(x), & \lim_{x \rightarrow -2^+} f(x), & \lim_{x \rightarrow -2^-} f(x), & \lim_{x \rightarrow 3} f(x), & \lim_{x \rightarrow 3^+} f(x), & \lim_{x \rightarrow 3^-} f(x) \\ = -5 & \text{DNE} & = -1 & = -\infty & = 4 & = 4 & = 4 \end{array}$$



2. Find each of the following limits for the function $f(x)$ shown below:

$$\begin{array}{lllllll} \lim_{x \rightarrow 0^-} f(x), & \lim_{x \rightarrow -1} f(x), & \lim_{x \rightarrow -1^+} f(x), & \lim_{x \rightarrow -1^-} f(x), & \lim_{x \rightarrow 1} f(x), & \lim_{x \rightarrow 1^+} f(x), & \lim_{x \rightarrow 1^-} f(x) \\ = 0 & = 1 & = 1 & = 1 & = 4 & = 4 & = 4 \end{array}$$



Find the limits:

$$3. \lim_{x \rightarrow 1} 5 = \boxed{5}$$

$$4. \lim_{x \rightarrow -2} 5 = \boxed{5}$$

$$5. \lim_{x \rightarrow 4} x = \boxed{4}$$

$$6. \lim_{x \rightarrow \pi} x = \boxed{\pi}$$

$$7. \lim_{x \rightarrow 0} \pi = \boxed{\pi}$$

$$8. \lim_{x \rightarrow 5} x(x+2)(x-1) = 5 \cdot 7 \cdot 4 = \boxed{140}$$

$$9. \lim_{x \rightarrow 2} x^2 - 4x + 3 = 2^2 - 4(2) + 3 = \boxed{-1}$$

$$10. \lim_{x \rightarrow 2} x^{17} - 4x^{15} + 3 = 2^{17} - 4 \cdot 2^{15} + 3 = \boxed{3}$$

$$11. \lim_{x \rightarrow 2} \frac{x^2 - x}{2x + 1} \xrightarrow{\frac{2}{5}} = \boxed{\frac{2}{5}}$$

$$12. \lim_{x \rightarrow 2} \frac{x^2 - x}{x - 1} \xrightarrow{\frac{2}{1}} = \boxed{2}$$

$$13. \lim_{x \rightarrow 1} \frac{x^2 - x}{x - 1} \xrightarrow{\frac{0}{0}} = \lim_{x \rightarrow 1} \frac{x(x-1)}{x-1} = \boxed{1}$$

$$14. \lim_{x \rightarrow 1^+} \frac{x^4 - 1}{x - 1} \xrightarrow{\frac{0}{0}} = \lim_{x \rightarrow 1^+} \frac{(x-1)(x+1)(x^2+1)}{x-1} = \boxed{4}$$

$$15. \lim_{z \rightarrow -1} \frac{z^2 + 6z + 5}{z^2 - 3z - 4} \xrightarrow{\frac{0}{0}} = \lim_{z \rightarrow -1} \frac{(z+1)(z+5)}{(z-1)(z-4)} = \boxed{-\frac{4}{5}}$$

$$16. \lim_{s \rightarrow -1} \frac{2s^2 + s - 1}{s + 1} \xrightarrow{\frac{0}{0}} = \lim_{s \rightarrow -1} \frac{(s+1)(2s-1)}{s+1} = 2(-1) - 1 = \boxed{-3}$$

$$17. \lim_{t \rightarrow 2} \frac{t^3 + 3t^2 - 12t + 4}{t^3 - 4t} \xrightarrow{\frac{8+12-24+4}{8-8} = \frac{0}{0}} \text{ see below } \downarrow$$

$$18. \lim_{x \rightarrow 2^+} \frac{x^2}{x-1} \xrightarrow{\frac{4}{1}} = \boxed{4}$$

$$19. \lim_{x \rightarrow 1^+} \frac{x^2}{x-1} \xrightarrow{\frac{1}{0^+}} = \boxed{+\infty}$$

$$20. \lim_{x \rightarrow 1^-} \frac{x^2}{x-1} \xrightarrow{\frac{1}{0^-}} = \boxed{-\infty}$$

$$21. \lim_{x \rightarrow 1} \frac{x^2}{x-1} = \boxed{\text{DNE}} \because \lim_{x \rightarrow 1^+} \neq \lim_{x \rightarrow 1^-}$$

$$22. \lim_{s \rightarrow 3^+} \frac{s+3}{s^2-9} \xrightarrow{\frac{6}{0^+}} = \boxed{+\infty}$$

$$23. \lim_{s \rightarrow 3^-} \frac{s+3}{s^2-9} \xrightarrow{\frac{6}{0^-}} = \boxed{-\infty}$$

$$24. \lim_{s \rightarrow 3} \frac{s+3}{s^2-9} = \boxed{\text{DNE}}$$

$$25. \lim_{t \rightarrow 4} \frac{3-t}{t^2-2t-8} \xrightarrow{\frac{-1}{0}} = \lim_{t \rightarrow 4} \frac{3-t}{(t-4)(t+2)} \text{ see below } \downarrow$$

$$26. \lim_{z \rightarrow 4} \frac{z-4}{\sqrt{z}-2} \xrightarrow{\frac{0}{0}} = \lim_{z \rightarrow 4} \frac{(z-4)(\sqrt{z}+2)}{\sqrt{z}-2} = \boxed{4}$$

$$27. \lim_{x \rightarrow 0} \frac{\sqrt{x+9}-3}{x} \cdot \frac{\sqrt{x+9}+3}{\sqrt{x+9}+3} = \lim_{x \rightarrow 0} \frac{x+9-9}{x(\sqrt{x+9}+3)} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

$$28. \lim_{x \rightarrow 0} \frac{\sqrt{x^2+9}-3}{x} \text{ see below } \downarrow$$

$$17. \begin{array}{r} t^2 + 5t - 2 \\ t-2 \overline{) t^3 + 3t^2 - 12t + 4} \\ \underline{t^3 - 2t^2} \\ 5t^2 - 12t \\ \underline{5t^2 - 10t} \\ -2t + 4 \\ \underline{-2t + 4} \\ 0 \end{array}$$

$$\therefore = \lim_{t \rightarrow 2} \frac{(t-2)(t^2+5t-2)}{t(t-2)(t+2)}$$

$$= \frac{2^2+5(2)-2}{2(4)} = \frac{12}{8} = \boxed{\frac{3}{2}}$$

$$25. \lim_{t \rightarrow 4^+} \frac{3-t}{(t-4)(t+2)} \xrightarrow{\frac{-1}{0^+ \cdot 6}} = -\infty$$

$$\lim_{t \rightarrow 4^-} \frac{3-t}{(t-4)(t+2)} \xrightarrow{\frac{-1}{0^- \cdot 6}} = +\infty$$

$$\therefore \lim_{t \rightarrow 4} \frac{3-t}{(t-4)(t+2)} = \boxed{\text{DNE}}$$

$$28. \lim_{x \rightarrow 0} \frac{\sqrt{x^2+9}-3}{x} \cdot \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3}$$

$$= \lim_{x \rightarrow 0} \frac{x^2+9-9}{x(\sqrt{x^2+9}+3)}$$

$$= \lim_{x \rightarrow 0} \frac{x}{\sqrt{x^2+9}+3} \xrightarrow{\frac{0}{6}} = \boxed{0}$$