

Homework Set 3: Solutions

1. $\lim_{x \rightarrow \infty} 5 = \boxed{5}$ (by axiom $\because 5$ is a constant)

2. $\lim_{x \rightarrow -\infty} x = \boxed{-\infty}$

3. $\lim_{x \rightarrow +\infty} x^2 = \boxed{+\infty}$

4. $\lim_{x \rightarrow +\infty} \sqrt{x} = \boxed{+\infty}$

5. $\lim_{x \rightarrow +\infty} \pi = \boxed{\pi}$ ($\because \pi$ is just a constant)

6. $\lim_{x \rightarrow -\infty} x(x+2)(x-1) \rightarrow -\infty \cdot -\infty \cdot -\infty = \boxed{-\infty}$

7. $\lim_{x \rightarrow \infty} x^2 - 4x + 3 = \lim_{x \rightarrow -\infty} x^2 = \boxed{+\infty}$

8. $\lim_{x \rightarrow -\infty} x^{17} - 4x^{15} + 3 = \lim_{x \rightarrow -\infty} x^{17} = \boxed{-\infty}$

9. $\lim_{x \rightarrow +\infty} \frac{x^2 - x}{2x + 1} \xrightarrow{\frac{\infty}{\infty}} = \lim_{x \rightarrow +\infty} \frac{x^2 - x}{2x + 1} \cdot \frac{1/x^2}{1/x^2}$
 $= \lim_{x \rightarrow +\infty} \frac{\cancel{x^2} - \cancel{x}}{\frac{2\cancel{x}}{x^2} + \frac{1}{x^2}}$
 $= \lim_{x \rightarrow +\infty} \frac{1 - \cancel{1/x} \xrightarrow{0}}{2\cancel{1/x} \xrightarrow{0} + \cancel{1/x^2} \xrightarrow{0}} \xrightarrow{\frac{1}{0}} = \boxed{+\infty}$

or just work with highest powers:

$\lim_{x \rightarrow +\infty} \frac{x^2 - x}{2x + 1} = \lim_{x \rightarrow +\infty} \frac{\cancel{x^2}}{2\cancel{x}} = \lim_{x \rightarrow +\infty} \frac{x}{2} \xrightarrow{\frac{+\infty}{2}} = \boxed{+\infty}$

10. $\lim_{s \rightarrow -\infty} \frac{2s^2 + s - 1}{s + 1} = \lim_{s \rightarrow -\infty} \frac{2\cancel{s^2}}{\cancel{s} + 1} \xrightarrow{\frac{-\infty}{+1}} = \boxed{-\infty}$

11. $\lim_{z \rightarrow +\infty} \frac{z^2 + 6z + 5}{z^2 - 3z - 4} = \lim_{z \rightarrow +\infty} \frac{\cancel{z^2} + 6\cancel{z} + 5}{\cancel{z^2} - 3\cancel{z} - 4} = \lim_{z \rightarrow +\infty} 1 = \boxed{1}$

$$12. \lim_{s \rightarrow +\infty} \frac{s^2 - 9}{s + 3} = \lim_{s \rightarrow +\infty} \frac{s}{1} = +\infty$$

13. $\lim_{t \rightarrow +\infty} \frac{4-t^3}{t^3-4t} = \lim_{t \rightarrow +\infty} \frac{-3t^2}{3t^2-4} = \lim_{t \rightarrow +\infty} \frac{-3}{3-\frac{4}{t^2}} = -1$

14. $\lim_{x \rightarrow +\infty} \sqrt{\frac{1 + 3x^2}{2 + 3x - 4x^2}} = \lim_{x \rightarrow +\infty} \sqrt{\frac{1 + 3x^2}{-4x^2}} = \sqrt{\frac{1}{-4}} = \sqrt{-\frac{1}{4}} = \frac{i}{2}$

or just working with highest powers:

$$= \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{3x^2}} = \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{3} |x|} = \frac{-1}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$$16. \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+7}}{5-x} = \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2}}{-x} = \lim_{x \rightarrow -\infty} \frac{\sqrt{3}|x|}{-x} = \frac{\sqrt{3}}{1}$$

$$17. \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 + 2x}}{x^2 - 1} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4}}{x^2} = \lim_{x \rightarrow +\infty} \frac{x^2}{x^2} = \boxed{1}$$

$$18. \lim_{x \rightarrow +\infty} \sqrt{x^2+3} - x = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+3} + x}{\sqrt{x^2+3} - x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+3} + x}{x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{1+\frac{3}{x^2}} + 1}{1} = 2$$

$$19. \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 3x} - x}{\sqrt{x^2 - 3x} + x} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 3x} - x}{\sqrt{x^2 - 3x} + x} \cdot \frac{\sqrt{x^2 - 3x} + x}{\sqrt{x^2 - 3x} + x} = \lim_{x \rightarrow \infty} \frac{x^2 - 3x - x^2}{x^2 - 3x + x^2 + x} = \lim_{x \rightarrow \infty} \frac{-3x}{2x^2 - 2x} = \lim_{x \rightarrow \infty} \frac{-3}{2x - 2} = -\frac{3}{\infty} = 0$$

$$= \frac{-3x + \sqrt{x^2(1-3/x) + x}}{-3x}$$

$$= \lim_{x \rightarrow -3} \frac{-3x}{x(\sqrt{1-3/x} + 1)}$$

$$= \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$20. \lim_{x \rightarrow +\infty} \frac{\sqrt{x+9} - 3}{x} \xrightarrow[\rightarrow \infty]{\rightarrow \infty} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\cancel{x/x^1}^0 + 9/\cancel{x^1}^0} - 3/\cancel{x^1}^0}{\cancel{x/x^1}^1} \xrightarrow[\rightarrow 1]{\rightarrow 0} = \boxed{0}$$

$$21. \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+9} - 3}{x} \xrightarrow[\rightarrow \infty]{\rightarrow \infty} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\cancel{x^2/x^2}^1 + 9/\cancel{x^2}^0} - 3/\cancel{x^1}^0}{\cancel{x/x^1}^1}$$

$$= \lim_{x \rightarrow +\infty} \frac{\sqrt{1}}{1} = \boxed{1}$$