

Homework Set 4: Continuity

Locate and classify the discontinuities (if any) in the following functions:

1. $f(x) = 3x^4 - x^2 + 6$

9. $f(x) = \frac{3x}{2x^2 + x}$

2. $f(x) = (x^3 - 2x + 1)^{12}$

10. $f(x) = \frac{4}{x} + \frac{x+2}{x^2-4}$

3. $f(x) = \sqrt{4-x}$

11. $f(x) = \begin{cases} 2x+2, & x \leq 4 \\ 6 + \frac{16}{x}, & x > 4 \end{cases}$

4. $f(x) = \sqrt[3]{x+8}$

5. $f(x) = \frac{1}{x^2+1}$

12. $f(x) = \begin{cases} 2x+1, & x \leq 4 \\ 8 + \frac{16}{x}, & x > 4 \end{cases}$

6. $f(x) = \frac{x+1}{x^2+1}$

7. $f(x) = \frac{x+1}{x^2-1}$

13. $f(x) = \begin{cases} \frac{2}{x-1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$

8. $f(x) = \frac{x^2 + 3x + 2}{x^2 + x - 2}$

Find a value of k that will make the function continuous everywhere:

14. $f(x) = \begin{cases} 7x-2, & x \leq 1 \\ kx^2, & x > 1 \end{cases}$

15. $f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x+k, & x > 2 \end{cases}$

16. $f(x) = \begin{cases} 9-x^2, & x \geq -3 \\ \frac{k}{x^2}, & x < -3 \end{cases}$

A function f has a removable discontinuity at c if $\lim_{x \rightarrow c} f(x)$ exists, but f is not continuous at c .

Such a discontinuity is “removable” because by redefining the value of f at c the discontinuity can be removed. Find the values of x (if any) at which f is not continuous and determine whether each discontinuity is removable.

17. $f(x) = \frac{|x|}{x}$

18. $f(x) = \frac{x^2 + 3x}{x + 3}$

19. $f(x) = \frac{x - 2}{|x| - 2}$

20. $f(x) = \frac{x^2 - 4}{x^3 - 8}$

21. $f(x) = \begin{cases} 3x^2 + 5, & x \neq 1 \\ 6, & x = 1 \end{cases}$

Applications of the Intermediate Value Theorem:

22. Show that the equation $x^3 + x^2 - 2x = 1$ has at least one solution in the interval $[-1, 1]$

23. Find the x -intercepts of $y = x^4 + x - 1$ with an error of at most 0.05.

24. Find the roots of the equation $5 - x - x^4 = 0$ to two decimal-place accuracy.

25. Use the fact that $\sqrt{5}$ is a solution of $x^2 - 5 = 0$ to approximate $\sqrt{5}$ with an error of at most 0.005.