

Homework Set 4 Solutions

1) $f(x) = 3x^4 - x^2 + 6$ has no discontinuities $\therefore$ it is a polynomial.

2) $f(x) = (x^3 - 2x + 1)^{12}$ has no discontinuities $\therefore$ if we expanded it we would get a polynomial.

3) $f(x) = \sqrt{4-x} = \sqrt{-x+4} = \sqrt{-(x-4)}$

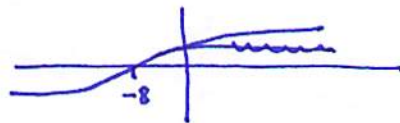
Let's look at it:



We can see it is: continuous for $x < 4$

discontinuous at $x = 4$ $\therefore \lim_{x \rightarrow 4} \sqrt{4-x}$ DNE
undefined for $x > 4$

4) $f(x) = \sqrt[3]{x+8}$ continuous for all x :



5) $f(x) = \frac{1}{x^2+1}$ continuous except where $x^2+1=0$ so $x^2+1 \leq 0$
 $x^2 = -1$

No real solution
 $\therefore$ continuous for all x .

6) $f(x) = \frac{x+1}{x^2+1}$ same story as above $\therefore$ continuous for all x .

7) $f(x) = \frac{x+1}{x^2-1}$ continuous except where $x^2-1=0 \Rightarrow x = \pm 1$

at $x=+1$, $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x+1}{x^2-1} \xrightarrow{\frac{2}{0}} = \text{DNE}$ asymptote

at $x=-1$, $\lim_{x \rightarrow -1} \frac{x+1}{x^2-1} \xrightarrow{\frac{0}{0}} = \lim_{x \rightarrow -1} \frac{x+1}{(x+1)(x-1)} = -\frac{1}{2}$ hole

8) $f(x) = \frac{x^2+3x+2}{x^2+x-2}$ $x^2+x-2 \leq 0$ $(x+2)(x-1)=0$
 $x=-2$ $x=+1$

at $x=-2$ $\lim_{x \rightarrow -2} \frac{x^2+3x+2}{x^2+x-2} \xrightarrow{\frac{-8}{0}} = \text{DNE}$ asymptote

at $x=1$ $\lim_{x \rightarrow 1} \frac{x^2+3x+2}{x^2+x-2} \xrightarrow{\frac{6}{0}} = \text{DNE}$ asymptote

$$9) f(x) = \frac{3x}{2x^2+x}$$

$$2x^2+x \stackrel{!}{=} 0$$

$$x(2x+1) = 0$$

$$x=0, 2x+1=0$$

$$x=-1/2$$

$$\text{at } x=0 \quad \lim_{x \rightarrow 0} \frac{3x}{2x^2+x} \xrightarrow{x \rightarrow 0} \frac{0}{0} \Rightarrow \text{hole}$$

$$\text{at } x=-1/2 \quad \lim_{x \rightarrow -1/2} \frac{3x}{2x^2+x} \xrightarrow{x \rightarrow -1/2} \frac{-3/2}{0} \Rightarrow \text{asymptote}$$

$$10) f(x) = \frac{4}{x} + \frac{x+2}{x^2-4}$$

$$\text{Discontinuity}$$

$$x=0$$

$$\text{lim form}$$

$$\xrightarrow{x \rightarrow 0} \frac{\infty}{0}$$

classification
asymptote

$$x=2$$

$$\xrightarrow{x \rightarrow 2} \frac{1}{0}$$

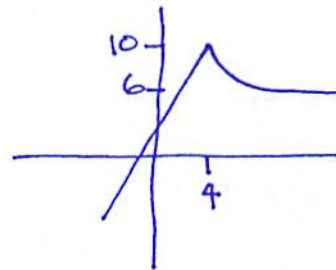
asymptote

$$x=-2$$

$$\xrightarrow{x \rightarrow -2} \frac{0}{0}$$

hole

$$11) f(x) = \begin{cases} 2x+2, & x \leq 4 \\ 6 + \frac{16}{x}, & x > 4 \end{cases}$$



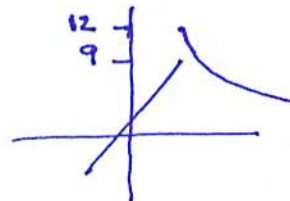
$x \leq 4$: $2x+2$ is continuous on this domain

$x > 4$: $6 + 16/x$ " " " " "

$x=4$: there is no "gap" here so no discontinuity

$\therefore f(x)$ is continuous for all x .

$$12) f(x) = \begin{cases} 2x+1, & x \leq 4 \\ 8 + \frac{16}{x}, & x > 4 \end{cases}$$

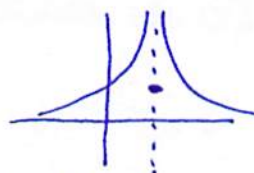


As above the individual pieces are continuous on their domains.

Unlike in 11) there is a gap at the boundary $x=4$

$\therefore$ there is a gap discontinuity at $x=4$.

$$13) f(x) = \begin{cases} 2/x - 1, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



Discontinuous at $x=1$: Displaced value & asymptote

$$14) f(x) = \begin{cases} 7x - 2, & x \leq 1 \\ kx^2, & x > 1 \end{cases}$$

To be continuous we need $7x - 2$ and kx^2 to meet and connect when $x = 1$

$$\text{so: } 7(1) - 2 \stackrel{!}{=} k(1)^2$$

$$\boxed{5 = k}$$

$$15) f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x + k, & x > 2 \end{cases}$$

$$k \cdot 2^2 \stackrel{!}{=} 2 \cdot 2 + k$$

$$4k = 4 + k$$

(Rationalise as above!)

$$3k = 4$$

$$\boxed{k = 4/3}$$

$$16) f(x) = \begin{cases} 9 - x^2, & x \geq -3 \\ k/x^2, & x < -3 \end{cases}$$

$$9 - (-3)^2 \stackrel{!}{=} \frac{k}{(-3)^2}$$

$$\boxed{0 = k}$$

$$17) f(x) = \frac{|x|}{x} \text{ has a gap at } x=0 \therefore \text{not removable } (\because \lim_{x \rightarrow 0} \text{ DNE})$$

$$18) f(x) = \frac{x^2 + 3x}{x+3} = \frac{x(x+3)}{x+3} \text{ has a hole at } x=-3 (\because \lim_{x \rightarrow -3} \text{ has form } \frac{0}{0})$$

$\therefore$ removable.

$$19) f(x) = \frac{x-2}{|x|-2} \text{ has discontinuities at } x = \pm 2$$

at $x = +2$: $\lim_{x \rightarrow 2}$ has form $\frac{0}{0}$ so hole and $\therefore$ removable.

at $x = -2$: $\lim_{x \rightarrow -2}$ has form $\frac{-2-2}{-2-2} = \frac{-4}{-4} = 1$ so asymptote

and $\therefore$ not removable.

$$20) f(x) = \frac{x^2 - 4}{x^3 - 8}$$

at $x = 2$: $\lim_{x \rightarrow 2}$ has form $\frac{0}{0}$ so hole and $\therefore$ removable.

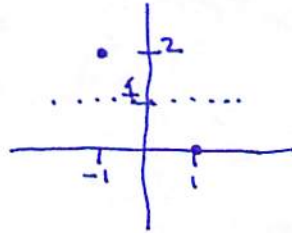
21) $f(x) = \begin{cases} 3x^2 + 5, & x \neq 1 \\ 6, & x = 1 \end{cases}$ at $x=1$, $f(x)=6$ but $\lim_{x \rightarrow 1} 3x^2 + 5 = 8$

$\therefore$ this is a displaced value & $\therefore$ removable

22) $x^3 + x^2 - 2x = 1$

at $x = -1$ we have $(-1)^3 + (-1)^2 - 2(-1) = 2$

at $x = +1$ we have $1^3 + 1^2 - 2(1) = 0$



Since $x^3 + x^2 - 2x$ is a polynomial and $\therefore$ continuous and $\therefore$ cannot get from $(-1, 2)$ to $(1, 0)$ without crossing the dotted line at $y = 1$.

$\therefore$ by the IVT there will be at least one solution on $(-1, 1)$.

23) $y = x^4 + x - 1$

x	y
0	-1
1	1
0.5	-0.4375
0.75	0.066406
0.625	-0.22241
0.6875	-0.0891
0.71875	-0.01437
0.734375	0.025227
0.726563	0.005234

x	y
0	-1
-1	-1
-2	13
-1.5	2.5625
-1.25	0.191406
-1.125	-0.52319
-1.1875	-0.19896
-1.21875	-0.01248

more than accurate enough

24) $5 - x - x^4 = 0$ "two decimal-place accuracy"

x	Y
0	5
-1	5
-2	-9
-1.5	1.4375
-1.75	-2.62891
-1.625	-0.3479
-1.5625	0.602036
-1.59375	0.141952
-1.60938	-0.09918
-1.60156	0.022325

2nd decimal place not changing so we can stop

x	Y
1	3
2	-13
1.5	-1.5625
1.25	1.308594
1.375	0.050537
1.4375	-0.70753
1.40625	-0.31...
1.390625	-0.13...
1.382813	-0.039...
1.379883	-0.00539

1.38 will be accurate to 2 d.p.

25) $x^2 - 5 = 0$

x	$x^2 - 5$
2	-1
3	4
2.5	1.25
2.25	0.0625
2.125	-0.48...
2.1875	-0.21...
2.234375	-0.007...
2.242188	0.02...
2.238281	0.009...
2.236328	0.001...
2.235352	-0.0032

} Since the solution is between these two we know it is 2.2358 ± 0.0005 .