

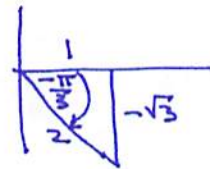
Homework Set 5: Solutions

- 1) $\lim_{x \rightarrow +\infty} \sin \frac{1}{x} \rightarrow \sin \frac{1}{\rightarrow \infty} \rightarrow \sin 0 = 0$
- 2) $\lim_{\theta \rightarrow 0} \frac{\sin 5\theta}{\theta} \times \frac{5}{5} = \lim_{\theta \rightarrow 0} 5 \cdot \frac{\sin 5\theta}{5\theta} = 5$
- 3) $\lim_{\theta \rightarrow 0} \frac{\theta^2 - 2\sin\theta}{\theta} = \lim_{\theta \rightarrow 0} \frac{\theta^2}{\theta} - \frac{2\sin\theta}{\theta} = \lim_{\theta \rightarrow 0} \theta - 2 \cdot \frac{\sin\theta}{\theta} = -2$
- 4) $\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} = \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \cdot \frac{1}{x} = +\infty$
- 5) $\lim_{x \rightarrow 0^+} \frac{\sin^2 x}{x} = \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \cdot \sin x = 0$
- 6) $\lim_{x \rightarrow 0^+} \frac{\sin x^2 \cdot x}{x} = \lim_{x \rightarrow 0^+} \frac{\sin x^2}{x^2} = 0$
- 7) $\lim_{x \rightarrow 0} \frac{\tan 5x}{\sin 3x} = \lim_{x \rightarrow 0} \frac{\sin 5x / \cos 5x}{\sin 3x} = \lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 3x} \cdot \frac{1}{\cos 5x}$
 $= \lim_{x \rightarrow 0} \frac{\frac{\sin 5x}{5x} \cdot \frac{5}{3} \cdot \frac{1}{\cos 5x}}{\frac{\sin 3x}{3x}} = \lim_{x \rightarrow 0} \frac{5 \cdot \frac{\sin 5x}{5x}}{3 \cdot \frac{\sin 3x}{3x}} \cdot \frac{1}{\cos 5x} = \frac{5}{3}$
- 8) $\lim_{x \rightarrow 0^+} \frac{\sin x}{3\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \lim_{x \rightarrow 0^+} \frac{\sin x}{3} \cdot \frac{\sqrt{x}}{x} = 0$
- 9) $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos^2 x} = \lim_{x \rightarrow 0} \frac{x^2}{\sin^2 x} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \rightarrow 0} \frac{1}{\frac{\sin^2 x}{x^2}} = \lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{x} \cdot \frac{\sin x}{x}} = 1$
- 10) $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \lim_{x \rightarrow 0} \frac{x^2 (1 + \cos x)}{1 - \cos^2 x} = \lim_{x \rightarrow 0} \frac{1 + \cos x}{\frac{1 - \cos^2 x}{x^2}} = 2$
- 11) $\lim_{\theta \rightarrow 0} \frac{\sin^2 \theta}{5\theta^2} = \lim_{\theta \rightarrow 0} 5 \left(\frac{\sin \theta}{\theta} \right)^2 = 5$

$$\begin{aligned}
 12) \lim_{h \rightarrow 0} \frac{\sin^2 3h + \tan^2 5h}{2h^2} &= \lim_{h \rightarrow 0} \frac{1}{2} \frac{\sin^2 3h}{h^2} + \frac{1}{2} \frac{\tan^2 5h}{h^2} \\
 &= \lim_{h \rightarrow 0} \frac{3^2}{2} \frac{\sin^2 3h}{(3h)^2} + \frac{5^2}{2} \frac{\sin^2 5h}{(5h)^2} \cdot \frac{1}{\cos^2 5h} \\
 &= \frac{3^2}{2} + \frac{5^2}{2} = \frac{34}{2} = 17
 \end{aligned}$$

$$13) \lim_{x \rightarrow +\infty} \frac{\cos \frac{\pi x}{2-3x}}{2-3x} = \cos \left(\lim_{x \rightarrow +\infty} \frac{\pi x}{2-3x} \right) = \cos \left(-\frac{\pi}{3} \right) = \frac{1}{2}$$

Note: limit is not at 0!



$$14) \lim_{x \rightarrow 0^+} \sin \frac{1}{x} \rightarrow \sin(\rightarrow \infty) = \text{DNE due to oscillation.}$$

$$15) \lim_{\theta \rightarrow 0} \frac{\sin \theta}{1 - \cos \theta} \cdot \frac{1/\theta}{1/\theta} = \lim_{\theta \rightarrow 0} \frac{\sin \theta / \theta}{(1 - \cos \theta) / \theta} = \text{DNE}$$

$$\begin{aligned}
 16) \lim_{\theta \rightarrow 0} \frac{1 - \cos 3\theta}{\cos^2 5\theta - 1} \cdot \frac{1 + \cos 3\theta}{1 + \cos 3\theta} &= \lim_{\theta \rightarrow 0} \frac{1 - \cos^2 3\theta}{\cos^2 5\theta - 1} \cdot \frac{1}{1 + \cos 3\theta} \\
 &= \lim_{\theta \rightarrow 0} \frac{\sin^2 3\theta}{-\sin^2 5\theta} \cdot \frac{1}{1 + \cos 3\theta} \\
 &= \lim_{\theta \rightarrow 0} \frac{3^2 \frac{\sin^2 3\theta}{(3\theta)^2}}{-5^2 \frac{\sin^2 5\theta}{(5\theta)^2}} \cdot \frac{1}{1 + \cos 3\theta} \\
 &= -\frac{3^2}{5^2} \cdot \frac{1}{2} = -\frac{9}{50}
 \end{aligned}$$

$$17) \lim_{\theta \rightarrow 0} \frac{\theta}{\cos \left(\frac{\pi}{2} - \theta \right)} = \lim_{\theta \rightarrow 0} \frac{\theta}{\sin \theta} \cdot \frac{1/\theta}{1/\theta} = \lim_{\theta \rightarrow 0} \frac{1}{\frac{\sin \theta}{\theta}} = 1$$

$$18) \lim_{x \rightarrow 5} \frac{\sin(x-5)}{x^2 - 25} = \lim_{x \rightarrow 5} \frac{\sin(x-5)}{x-5} \cdot \frac{1}{x+5} = \frac{1}{10}$$

$$\begin{aligned}
 19) \lim_{x \rightarrow -2} \frac{\sin(x^2 + 3x + 2)}{x+2} &= \lim_{x \rightarrow -2} \frac{\sin[(x+2)(x+1)]}{x+2} \cdot \frac{x+1}{x+1} \\
 &= \lim_{x \rightarrow -2} \frac{\sin[(x+2)(x+1)]}{(x+2)(x+1)} \cdot (x+1)^{-1} \\
 &= -1
 \end{aligned}$$

20) $\lim_{x \rightarrow +\infty} x \sin \frac{1}{x}$ introduce $t = \frac{1}{x}$ so $x = \frac{1}{t}$ and $x \rightarrow +\infty \Rightarrow t \rightarrow 0$

then $\lim_{x \rightarrow +\infty} x \sin \frac{1}{x} = \lim_{t \rightarrow 0} \frac{1}{t} \sin t = \lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$

21) $\lim_{x \rightarrow -\infty} x(1 - \cos \frac{1}{x}) = \lim_{t \rightarrow 0} \frac{1}{t} (1 - \cos t) = \lim_{t \rightarrow 0} \frac{1 - \cos t}{t} = 0$

22) $\lim_{x \rightarrow \pi} \frac{\pi - x}{\sin x}$ introduce $t = \pi - x$ so $x \rightarrow \pi \Rightarrow t \rightarrow \pi - \pi = 0$
and note $\sin(\pi - x) = \sin x$

$= \lim_{t \rightarrow 0} \frac{t}{\sin t} = \lim_{t \rightarrow 0} \frac{1}{\frac{\sin t}{t}} = 1$

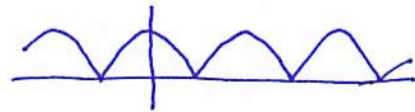
23) $f(x) = 1 - \sin x$ 1 & $\sin x$ are both continuous $\forall x \therefore f$ is too

24) $f(x) = \cos x - \sin x$ $\cos x$ & $\sin x$ " " " " " " " " " "

25) $f(x) = \cot x = \frac{\cos x}{\sin x} \therefore$ continuous except where $\sin x = 0$ i.e. $x = \pm n\pi$

26) $f(x) = \csc x = \frac{1}{\sin x} \therefore$ " " " " " " " " " "

27) $f(x) = |\cos x|$ continuous $\forall x$



28) $f(x) = \frac{1}{1 + \sin x}$ continuous except where $1 + \sin x = 0$
 $\sin x = -1$

$x = \dots, \frac{5\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \dots$

29) $f(x) = \frac{1}{1 - \cos x}$ continuous except where $1 - \cos x = 0$
 $1 = \cos x \quad x = \dots, -2\pi, 0, 2\pi, \dots$

30) $f(x) = \frac{1}{1 + \sin^2 x}$ continuous $\forall x \because 1 + \sin^2 x \neq 0$ for any x
 $\because \sin^2 x \geq 0 \quad \forall x.$