

Homework Set 8: Average and Instantaneous Rates of Change

1. What is the average rate of change of $y = x^3$ over the interval $[1, 4]$? What is the instantaneous rate of change of y with respect to x when $x = 1$? When $x = 4$?

Avg. rate of change on $[1, 4] = \frac{\Delta y}{\Delta x} = \frac{4^3 - 1^3}{4 - 1} = \frac{63}{3} = 21$

Inst. rate of change when $x = 1$: $\left. \frac{dy}{dx} \right|_{x=1} = 3x^2 \Big|_{x=1} = 3(1)^2 = 3$

" " " " " $x = 4$: $\left. \frac{dy}{dx} \right|_{x=4} = 3x^2 \Big|_{x=4} = 3 \cdot 4^2 = 48$

2. What is the average rate of change of $y = \sin x$ over the interval $[0, \pi]$? What is the instantaneous rate of change of y with respect to x when $x = 0$? When $x = \pi$?

Avg: $\frac{\Delta y}{\Delta x} = \frac{\sin \pi - \sin 0}{\pi - 0} = \frac{0 - 0}{\pi} = 0$

Inst: $\left. \frac{dy}{dx} \right|_{x=0} = \cos x \Big|_{x=0} = \cos 0 = 1$

$\left. \frac{dy}{dx} \right|_{x=\pi} = \cos x \Big|_{x=\pi} = \cos \pi = -1$

3. The volume, V , of a balloon in cm^3 while it is being inflated is modeled as a function of time, t in seconds, by $V = 8\sqrt{t}$. What is the average rate at which the balloon inflates over the first 10 seconds? What is the average rate at which the balloon inflates during the first second? During the 10th second? What is the instantaneous rate of change of the balloon's volume after 10 seconds? What is the instantaneous rate of change of the balloon's volume at the instant the balloon begins inflating?

a) $\frac{\Delta V}{\Delta t} = \frac{8\sqrt{10} - 8\sqrt{0}}{10 - 0} = \frac{8\sqrt{10}}{10} = \frac{8}{\sqrt{10}} \approx 2.53 \frac{\text{cm}^3}{\text{s}}$

b) $\frac{\Delta V}{\Delta t} = \frac{8\sqrt{1} - 8\sqrt{0}}{1 - 0} = 8 \frac{\text{cm}^3}{\text{s}}$

c) $\frac{\Delta V}{\Delta t} = \frac{8\sqrt{10} - 8\sqrt{9}}{10 - 9} = 8(\sqrt{10} - \sqrt{9}) \approx 1.30 \frac{\text{cm}^3}{\text{s}}$

d) $\left. \frac{dV}{dt} \right|_{t=10} = 8 \cdot \frac{1}{2\sqrt{t}} \Big|_{t=10} = \frac{4}{\sqrt{t}} \Big|_{t=10} = \frac{4}{\sqrt{10}} \approx 1.26 \frac{\text{cm}^3}{\text{s}}$

e) $\left. \frac{dV}{dt} \right|_{t=0} = \frac{4}{\sqrt{t}} \Big|_{t=0} = \text{undefined} \quad \lim_{t \rightarrow 0^+} \frac{dV}{dt} = +\infty$