

Homework Set 10: Derivatives of Compositions (The Chain Rule)

Find $f'(x)$:

$$1. f(x) = (x^2 - x)^{11} \quad f'(x) = 11(x^2 - x)^{10} \cdot (2x - 1)$$

$$2. f(x) = \left(x - \frac{2}{x}\right)^{-3} \quad f'(x) = -3\left(x - \frac{2}{x}\right)^{-4} \cdot \left(1 + \frac{2}{x^2}\right)$$

$$3. f(x) = \frac{1}{(2x^3 - 3x + 1)^4} = (2x^3 - 3x + 1)^{-4} \quad f'(x) = -4(2x^3 - 3x + 1)^{-5} \cdot (6x^2 - 3)$$

$$4. f(x) = \sqrt{7x} \quad f'(x) = \frac{1}{2\sqrt{7x}} \cdot 7 = \frac{7}{2\sqrt{7x}}$$

$$5. f(x) = \sin(x^2) \quad f'(x) = \cos(x^2) \cdot 2x$$

$$6. f(x) = \cos \frac{2}{x} \quad f'(x) = -\sin\left(\frac{2}{x}\right) \cdot \left(-\frac{2}{x^2}\right) = \frac{2}{x^2} \sin\left(\frac{2}{x}\right)$$

$$7. f(x) = 3 \sin^2 4x \quad f'(x) = 3(2 \sin 4x \cdot \cos 4x \cdot 4) = 24 \sin 4x \cos 4x$$

$$8. f(x) = \sec^2 \sqrt{5x} = (\sec \sqrt{5x})^2 \quad f'(x) = 2 \sec \sqrt{5x} \cdot \sec \sqrt{5x} \tan \sqrt{5x} \cdot \frac{1}{2\sqrt{5x}} \cdot 5$$

$$9. f(x) = \sqrt{\sin 4x} \quad f'(x) = \frac{1}{2\sqrt{\sin 4x}} \cdot \cos 4x \cdot 4$$

$$10. f(x) = [x^3 - \sec(x^2 - 2x)]^{-5} \quad f'(x) = -5[x^3 - \sec(x^2 - 2x)]^{-6} \cdot (3x^2 - \sec(x^2 - 2x) \tan(x^2 - 2x) \cdot (2x - 2))$$

Find $\frac{dy}{dx}$:

$$11. y = x^2 \sin^3 4x \quad \frac{dy}{dx} = x^2 \cdot (3 \sin^2 4x \cdot \cos 4x \cdot 4) + \sin^3 4x \cdot (2x)$$

$$12. y = \sin(\sin x) \quad \frac{dy}{dx} = \cos(\sin x) \cdot \cos x$$

$$13. y = \sec^2(\sin 3x) \quad \frac{dy}{dx} = 2 \sec(\sin 3x) \cdot \sec(\sin 3x) \tan(\sin 3x) \cdot \cos 3x \cdot 3$$

$$14. y = \left(1 - \frac{2}{x}\right)^2 \sin^3 4x \quad \frac{dy}{dx} = \left(1 - \frac{2}{x}\right)^2 \cdot 3 \sin^2 4x \cdot \cos 4x \cdot 4 + \sin^3 4x \cdot 2 \left(1 - \frac{2}{x}\right)^1 \cdot \frac{2}{x^2}$$

$$15. y = \frac{(2x-1)^3}{(x^3+1)^6} \quad \frac{dy}{dx} = \frac{(x^3+1)^6 \cdot (3(2x-1)^2 \cdot 2) - (2x-1)^3 \cdot (6(x^3+1)^5 \cdot 3x^2)}{(x^3+1)^{12-7}} \\ = \frac{6(2x-1)^2 [(x^3+1) - 3x^2(2x-1)]}{(x^3+1)^7}$$

$$16. y = \sin^4(x^2 - \tan^3 \sqrt{x^3 - \cos x})$$

$$\frac{dy}{dx} = 4(\sin(x^2 - \tan^3 \sqrt{x^3 - \cos x}))^3 \cdot (2x - 3 \tan^2 \sqrt{x^3 - \cos x} \cdot \sec^2 \sqrt{x^3 - \cos x} \cdot \frac{1}{2\sqrt{x^3 - \cos x}} \cdot (3x^2 + \sin x))$$