

Homework Set 11: Implicit Differentiation

Find $\frac{dy}{dx}$ by implicit differentiation:

$$\frac{d}{dx}[x^3 + y^3] = \frac{d}{dx}[3xy^2] \Rightarrow 3x^2 + 3y^2 \frac{dy}{dx} = 3x \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot 3$$

$$1. \quad x^3 + y^3 = 3xy^2 \Rightarrow 3y^2 \frac{dy}{dx} - 6xy \frac{dy}{dx} = 3y^2 - 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3y^2 - 3x^2}{3y^2 - 6xy}$$

$$2. \quad x^2y + 2xy^3 + x = 4 \quad x^2 \frac{dy}{dx} + y \cdot 2x + 2x \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 2 + 1 = 0$$

$$x^2 \frac{dy}{dx} + 6xy^2 \frac{dy}{dx} = -1 - 2y^3 - 2xy \Rightarrow \frac{dy}{dx} = \frac{-1 - 2y^3 - 2xy}{x^2 + 6xy^2}$$

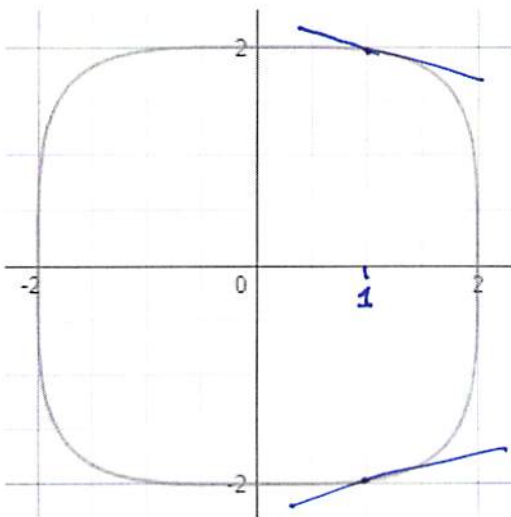
$$3. \quad \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 0 \quad \frac{-1}{2} x^{-3/2} - \frac{1}{2} y^{-3/2} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-\frac{1}{2} x^{-3/2}}{-\frac{1}{2} y^{-3/2}} = \frac{y^{3/2}}{x^{3/2}} = \left(\frac{y}{x}\right)^{3/2}$$

$x^{-1/2} + y^{-1/2} = 0$ →

$$4. \quad \sin(xy^2) = x \Rightarrow \cos(xy^2) \cdot (x \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot 1) = 1$$

$$2xy \cos(xy^2) \frac{dy}{dx} = 1 - y^2 \cos(xy^2) \Rightarrow \frac{dy}{dx} = \frac{1 - y^2 \cos(xy^2)}{2xy \cos(xy^2)}$$

5. Find the slopes of the tangent lines to Lamé's special quartic $x^4 + y^4 = 16$ when $x = 1$.



$$m_{\text{tan}} = \frac{dy}{dx}$$

$$x^4 + y^4 = 16 \Rightarrow 4x^3 + 4y^3 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-4x^3}{4y^3} = -\frac{x^3}{y^3}$$

$$x = 1 \Rightarrow 1^4 + y^4 = 16$$

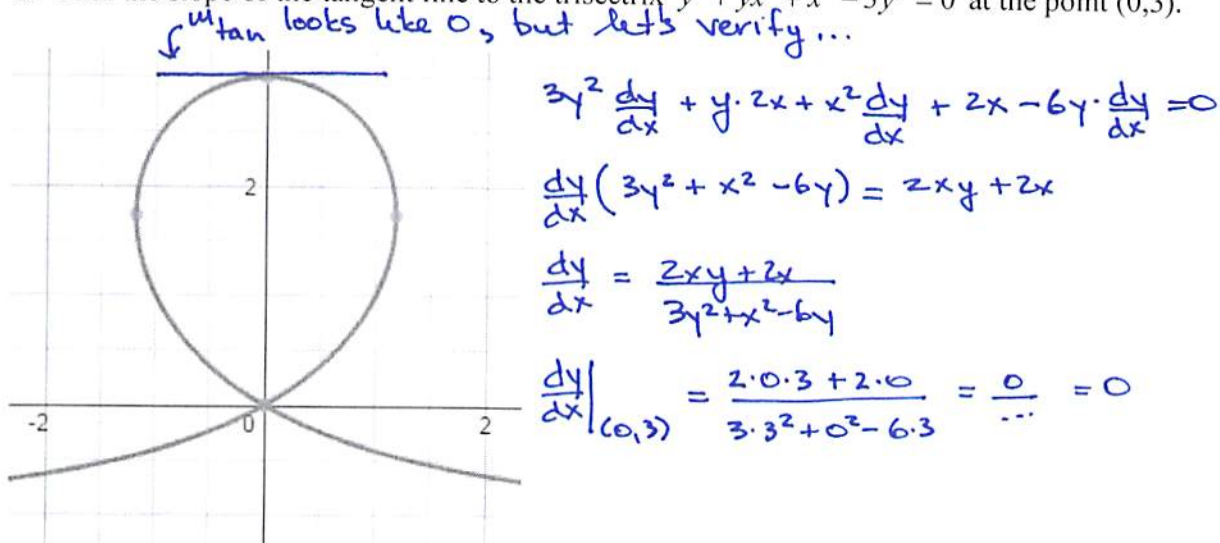
$$y^4 = 15$$

$$y = \pm \sqrt[4]{15}$$

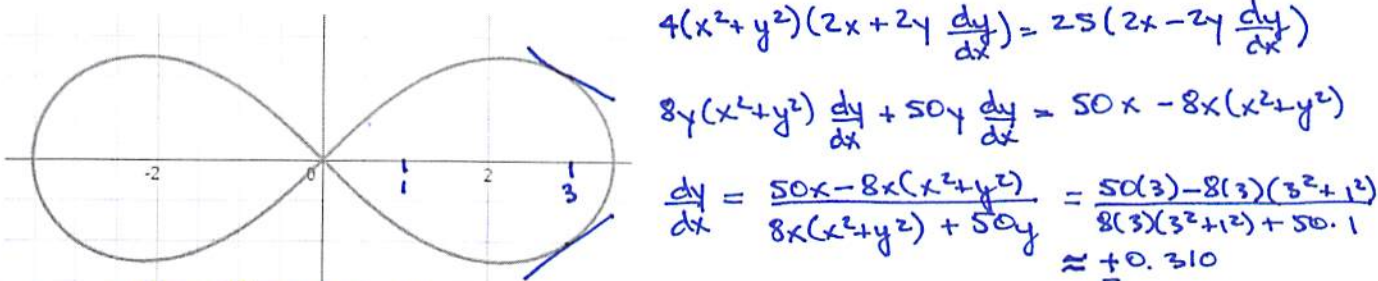
$$\therefore m_{\text{tan}} @ (1, \sqrt[4]{15}) = \frac{-1^3}{(\sqrt[4]{15})^3} = \frac{-1}{15^{3/4}}$$

$$m_{\text{tan}} @ (1, -\sqrt[4]{15}) = \frac{-1^3}{(-\sqrt[4]{15})^3} = \frac{1}{15^{3/4}}$$

6. Find the slope of the tangent line to the trisectrix $y^3 + yx^2 + x^2 - 3y^2 = 0$ at the point $(0,3)$.



7. Find the slopes of the tangent lines to the lemniscate $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ when $x = 3$.



$$x=3 \Rightarrow 2(9+y^2)^2 = 25(9-y^2)$$

$$2y^4 + 25y^2 + 36y - 63 = 0 \Rightarrow y = \pm 1$$

8. Find the slopes of the tangent lines to the four-cusped hypocycloid $x^{2/3} + y^{2/3} = 4$ when $x = -1$.

