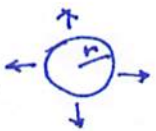
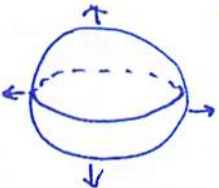


Homework Set 12 Solutions

1)  Given $\frac{dr}{dt} = 1.2 \frac{m}{s}$ Find $\left. \frac{dA}{dt} \right|_{t=10}$ $\therefore$ at $t=0$ $r=0$
so at $t=10$ $r=10 \cdot 1.2 = 12$

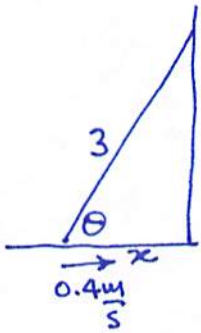
Sol'n $A = \pi r^2 \Rightarrow \frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} \Rightarrow \left. \frac{dA}{dt} \right|_{t=10} = \pi \cdot 2 \cdot 12 \cdot 10 = 240\pi \frac{m^2}{s}$

2)  Given $\frac{dV}{dt} = 1.2 \frac{m^3}{min}$ Find $\left. \frac{dr}{dt} \right|_{r=0.5}$

Sol'n $V = \frac{4}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3} \pi \cdot 3r^2 \cdot \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2} \frac{dV}{dt}$

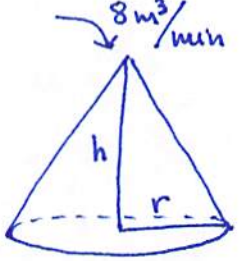
$\left. \frac{dr}{dt} \right|_{r=0.5} = \frac{1}{4\pi \cdot 0.5^2} \cdot 1.2 = \frac{1.2}{\pi} \frac{m}{min}$

3)  Given $\frac{dx}{dt} = -0.4 \frac{m}{s}$ Find $\left. \frac{d\theta}{dt} \right|_{x=0.5}$

Sol'n $\cos \theta = \frac{x}{3} \Rightarrow x = 3 \cos \theta \Rightarrow \frac{dx}{dt} = -3 \sin \theta \cdot \frac{d\theta}{dt}$

$\therefore \frac{d\theta}{dt} = \frac{-1}{3 \sin \theta} \frac{dx}{dt}$

$\left. \frac{d\theta}{dt} \right|_{x=0.5} = \frac{-1}{3 \cdot \frac{\sqrt{3^2 - x^2}}{3}} \cdot -0.4 = \frac{+0.4}{\sqrt{9 - 0.5^2}} \approx 0.135 \frac{rad}{s}$

4)  Given $\frac{dV}{dt} = 8 \frac{m^3}{min}$ Find $\frac{dh}{dt}$ for $h=2$ and $h=6$

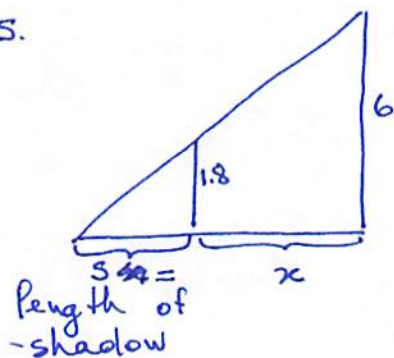
Sol'n $V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4}{\pi h^2} \frac{dV}{dt}$

$\left. \frac{dh}{dt} \right|_{h=2} = \frac{4}{\pi \cdot 2^2} \cdot 8 = \frac{8}{\pi} \frac{m}{min}$

$\left. \frac{dh}{dt} \right|_{h=6} = \frac{4}{\pi \cdot 6^2} \cdot 8 = \frac{8}{9\pi} \frac{m}{min}$

5.



Given $\frac{dx}{dt} = -1 \frac{m}{s}$ Find $\frac{ds}{dt}$

Solution Similar $\Delta s \Rightarrow \frac{s}{1.8} = \frac{x+s}{6}$

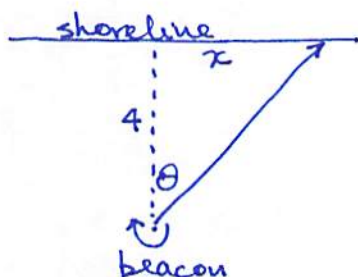
$$6s = 1.8x + 1.8s$$

$$4.2s = 1.8x$$

$$4.2 \frac{ds}{dt} = 1.8 \frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{1.8}{4.2} \frac{dx}{dt} = -\frac{3}{7} \frac{m}{s}$$

6.



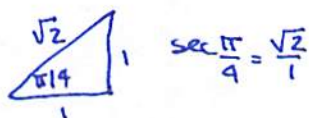
Given $\frac{d\theta}{dt} = \frac{2\pi \text{ rad}}{10 \text{ s}} = \frac{\pi}{5} \frac{\text{rad}}{\text{s}}$

Find $\frac{dx}{dt} \big|_{\theta=45^\circ}$

Soln $\tan \theta = \frac{x}{4} \Rightarrow x = 4 \tan \theta$

$$\frac{dx}{dt} = 4 \sec^2 \theta \cdot \frac{d\theta}{dt}$$

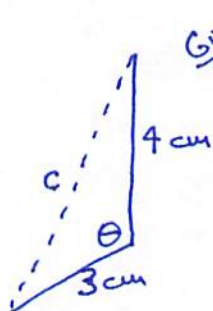
$$\frac{dx}{dt} \big|_{\theta=45^\circ} = 4 \sec^2(\pi/4) \cdot \frac{\pi}{5} = 4 \cdot (\sqrt{2})^2 \frac{\pi}{5} = \frac{8\pi}{5} \frac{\text{km}}{\text{s}}$$



7.

At 8 o'clock

$$\theta = \frac{1}{3} 2\pi = \frac{2\pi}{3}$$



Given

Known rate is rate at which hands move
minute hand: $\frac{1 \text{ rev}}{\text{hour}} = 2\pi \frac{\text{rad}}{\text{hr}}$

hour hand: $\frac{1 \text{ rev}}{12 \text{ hrs}} = \frac{2\pi}{12} \frac{\text{rad}}{\text{hr}} = \frac{\pi}{6} \frac{\text{rad}}{\text{hr}}$

$\therefore$ angle θ between them changes at $2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \frac{\text{rad}}{\text{hr}} = \frac{d\theta}{dt}$

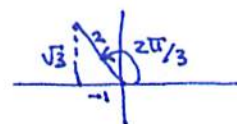
Find $\frac{dc}{dt} \big|_{\theta=\frac{2\pi}{3}}$

Soln $c^2 = 3^2 + 4^2 - 2(3)(4) \cos \theta$ (cosine law)

$$2c \frac{dc}{dt} = 0 + 0 + 24 \sin \theta \cdot \frac{d\theta}{dt}$$

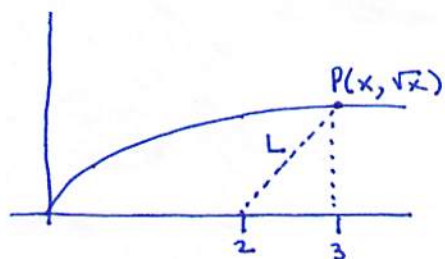
$$\frac{dc}{dt} = \frac{12 \sin \theta}{c} \frac{d\theta}{dt}$$

At 8 o'clock: $c = \sqrt{3^2 + 4^2 - 2(3)(4) \cos \frac{2\pi}{3}}$
 $= \sqrt{9 + 16 - 24 \cdot -1/2} = \sqrt{37}$



$$\therefore \frac{dc}{dt} = \frac{12 \cdot \sqrt{3}/2}{\sqrt{37}} \frac{11\pi}{6} = \frac{11\sqrt{3}\pi}{\sqrt{37}} \approx 3.13\pi \frac{\text{cm}}{\text{hr}}$$

8.



Given $\frac{dx}{dt} = 4$ Find $\frac{dL}{dt} \Big|_{x=3}$

Soln $L^2 = (x-2)^2 + (\sqrt{x})^2 = x^2 - 4x + 4 + x$
 $= x^2 - 3x + 4$

When $x=3$

$$L = \sqrt{(3-2)^2 + (\sqrt{3})^2}$$

$$= \sqrt{1+3}$$

$$= 2$$

$$2L \frac{dL}{dt} = 2x \frac{dx}{dt} - 3 \frac{dx}{dt} + 0$$

$$\frac{dL}{dt} = \frac{1}{2L} (2x-3) \frac{dx}{dt}$$

$$= \frac{1}{2 \cdot 2} (2 \cdot 3 - 3) \cdot 4 = 3 \frac{\text{units}}{\text{s}}$$

9.



$$V = \frac{4}{3} \pi r^3$$

$$S = 4\pi r^2$$

Given $\frac{dV}{dt} \propto 4\pi r^2$ Show $\frac{dr}{dt}$ is constant

$$= k \cdot 4\pi r^2$$

Soln $V = \frac{4}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3} \pi \cdot 3r^2 \frac{dr}{dt}$

$$\therefore \frac{dr}{dt} = \frac{1}{4\pi r^2} \frac{dV}{dt}$$

$$= \frac{1}{4\pi r^2} \cdot k \cdot 4\pi r^2$$

$$= k. \text{ Q.E.D.}$$