

Homework Set 15: Optimization

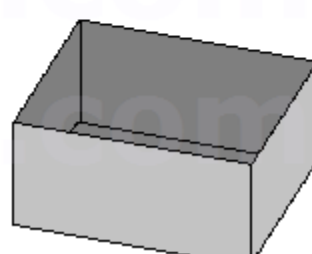
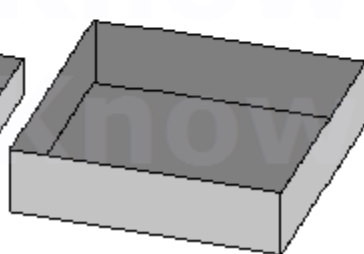
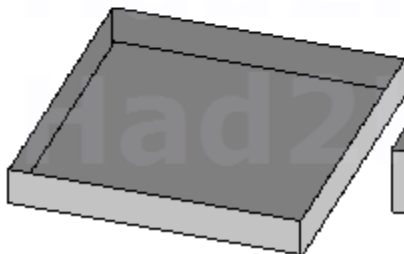
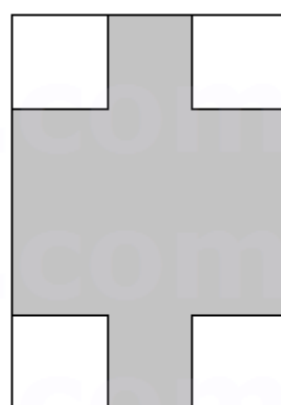
1. Find a number in the interval $\left[\frac{1}{2}, \frac{3}{4}\right]$ such that the sum of a number and its reciprocal is
 - a. As small as possible
 - b. As large as possible
2. A rectangular area of 3200 m^2 is to be fenced off. Two opposite sides will use fencing costing \$1 per m and the remaining sides will use fencing costing \$2 per m. What should the dimensions of the area be to minimize the cost?



3. An *open* cylindrical can is to hold 500 cm^3 of liquid. Find the height and radius that minimize the amount of material used.
4. A cone-shaped paper drinking cup is to hold 100 cm^3 of liquid. Find the height and radius of the cup that will require the least paper.
5. An orange juice can has a volume of $48\pi \text{ cm}^3$ and has metal ends and cardboard “sides”. The metal costs 3 times as much per cm^2 as the cardboard. What dimensions of the can will minimize the cost of material?



6. A metal water trough is to be built from a flat 2 m by 3 m sheet of metal by cutting squares from each corner and then folding the sides up to create a rectangular prism. What size squares should be cut to maximize the volume of the trough?



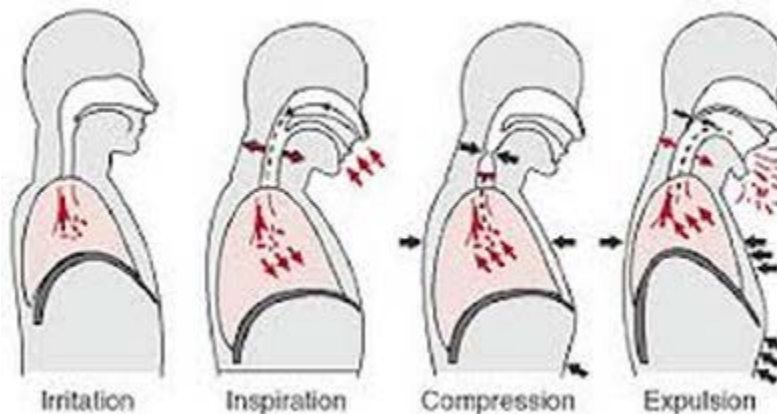
7. A fish cannery would like to minimize the amount of material it uses in its cans. What should the height and radius of the can be to minimize the amount of material used?



8. A small town newspaper sells for \$1 per copy. The cost of producing x copies is $C(x) = 70 + 0.001x + 0.00005x^2$ dollars.
- What does the 70 in the cost function represent?
 - What is the revenue function for this operation?
 - What is the profit function for this operation?
 - What number of copies will maximize profit? At this point what is the marginal profit? The marginal revenue?
 - Suppose the newspaper is taken over by a community group and run as a non-profit venture whose goal is to maximize readership while breaking even. What number of copies would represent their optimum? At this point what is the marginal profit? The marginal revenue?
9. A chemical manufacturer sells sulfuric acid in bulk at a price of \$100 per unit.
- If the daily total production cost in dollars for x units is $C(x) = 100,000 + 50x + 0.0025x^2$ and if the daily production capacity is at most 7,000 units, how many units of sulfuric acid should be manufactured and sold daily to maximize the profit?
 - What is the marginal profit when the production level is 7,000 units?
 - Would it benefit the manufacturer to expand daily production capacity?
10. The cost of controlling emissions often rises rapidly as the amount of emissions reduced increases, i.e. it is cheap and easy to find the first reductions, but becomes increasingly difficult and expensive to find further reductions. Here is a possible model for cost, $C(r) = 40 + 10r^2$ where r is the reduction in emissions in pounds of pollutant per day and C is the daily cost in \$ of this reduction.



- a. If a firm is currently reducing its emissions by 10 pounds per day, what is the marginal cost of reducing emission further?
 - b. Government clean air subsidies to the firm are based on the formula $S(r) = 500r$ dollars where r is again the daily reduction and S is the subsidy in dollars per day. Will this subsidy result in a reduction in emissions? Why or why not? If it will, how large a reduction?
11. Two years ago your orange orchard contained 50 trees and yielded 75 bags of oranges. Last year you removed 10 of the trees and noticed that the total yield increased to 80 bags. Assuming that the yield per tree depends linearly on the number of trees in the orchard, what should you do this year to maximize your total yield?
12. What is the optimum radius of the trachea when coughing? The goal is to create a strong outward force in the throat to clear it.



For this problem you should perform the following experiment:

Hold your hand about 10 cm from your mouth and blow on it (a) with your lips compressed almost closed but with a small stream of air escaping, (b) with your mouth wide open, and (c) with your lips adjusted to create the largest force on your hand.

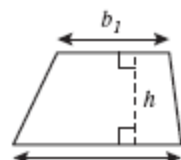
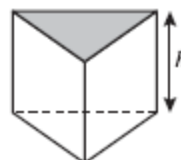
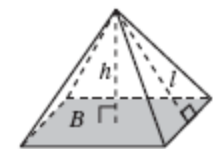
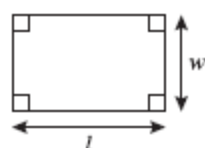
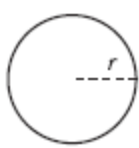
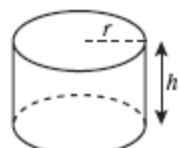
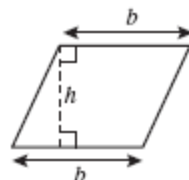
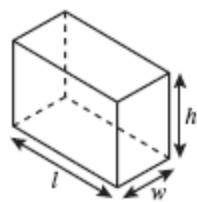
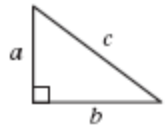
With (a) your lips almost closed there is a high pressure causing rapid air flow but a small stream of air and little force. With (b) your mouth wide open there is a large stream of air but with little pressure so that air flow is slow. The largest force (c) is created with an intermediate opening of your lips where there is a notable pressure and rapid flow of substantial volume of air.

Let R be the normal radius of the trachea and r be the reduced radius when coughing. Assume that the velocity of air flow through the trachea is proportional to pressure difference across the trachea and that the pressure difference is proportional to the constriction of the trachea. Assume that the mass of air flow is proportional to the area of the trachea, and that its cross-section is circular. Finally the momentum, M , of the air flow is the product of the mass of the air flow and its velocity.

We would like to find the value of r that maximizes M .

- a) If R is the normal radius of the trachea and r is the reduced radius write an expression for the constriction.
- b) Use your expression from a) to translate the sentence “Assume that the velocity of air flow through the trachea is proportional to pressure difference across the trachea and that the pressure difference is proportional to the constriction of the trachea.” into an equation for the velocity v of the air flow.
- c) What is an expression for the area of a circle?
- d) use your expression from c) to translate the sentence “Assume that the mass of air flow is proportional to the area of the trachea, and that its cross-section is circular.” Into an equation for, m , the mass of the air flow.
- e) Combine your equations from b) and c) to translate the sentence “Finally the momentum, M , of the air flow is the product of the mass of the air flow and its velocity.” into an equation for M in terms of r .
- f) Find the value of r that maximizes M by taking the derivative of M with respect to r setting it equal to 0 and solving for r .
- g) Since we don’t have an exact numerical value for R our answer in f) will be in terms of R . Interpret that expression.

Geometric Formulas

 $A = \frac{1}{2}bh$	 $A = \frac{1}{2}h(b_1 + b_2)$	 $V = Bh$ $L.A. = hp$ $S.A. = L.A. + 2B$	 $V = \frac{1}{3}Bh$ $L.A. = \frac{1}{2}lp$ $S.A. = L.A. + B$
 $A = lw$ $p = 2(l + w)$	 $A = \pi r^2$ $C = 2\pi r$	 $V = \pi r^2h$ $L.A. = 2\pi rh$ $S.A. = 2\pi r(h + r)$	 $V = \frac{4}{3}\pi r^3$ $S.A. = 4\pi r^2$
 $A = bh$	 $V = lwh$ $S.A. = 2lw + 2lh + 2wh$	 $V = \frac{1}{3}\pi r^2h$ $L.A. = \pi rl$ $S.A. = \pi r(l + r)$	 $c^2 = a^2 + b^2$

Finance formulas

Revenue = unit sales $\times$ price per unit

Cost = units produced $\times$ cost per unit

Profit = Revenue – Cost

Marginal Cost = Change in cost to produce one more unit

Marginal Revenue = Change in revenue by increasing sales by one unit

Marginal Profit = Change in profit due to producing and selling one more unit