

Homework Set 16: The Indefinite Integral

Evaluate the following integrals:

$$1. \int x^6 dx = \frac{1}{7} x^7 + C$$

$$2. \int x^{3/7} dx = \frac{7}{10} x^{10/7} + C \quad \because \frac{3}{7} + 1 = \frac{3}{7} + \frac{7}{7} = \frac{10}{7}$$

$$3. \int x^2 \sqrt{x} dx = \int x^2 x^{1/2} dx = \int x^{5/2} dx = \frac{2}{7} x^{7/2} + C$$

$$4. \int \left[3z - \frac{4}{3z^7} \right] dz = 3 \int z dz - \frac{4}{3} \int z^{-7} dz = 3 \cdot \frac{z^2}{2} - \frac{4}{3} \left(\frac{-1}{6} \right) z^{-6} + C = \frac{3z^2}{2} + \frac{2}{9} z^{-6} + C$$

$$5. \int [x^{-4} - 5x^{1/3} - 1] dx = -\frac{x^{-3}}{3} - 5 \left(\frac{3}{4} \right) x^{4/3} - x + C = -\frac{x^{-3}}{3} - \frac{15x^{4/3}}{4} + C$$

$$6. \int [3 \sin x - 5 \sec^2 x] dx = 3 \int \sin x dx - 5 \int \sec^2 x dx = 3(-\cos x) - 5 \tan x + C = -3 \cos x - 5 \tan x + C$$

$$7. \int y(2 - y^2) dy = \int [2y - y^3] dy = y^2 - \frac{y^4}{4} + C$$

$$8. \int x^{1/3} (1+x)^2 dx = \int x^{1/3} (1+2x+x^2) dx = \int (x^{1/3} + 2x^{4/3} + x^{7/3}) dx = \frac{3}{4} x^{4/3} + \frac{6}{7} x^{7/3} + \frac{3}{10} x^{10/3} + C$$

correction $\rightarrow$

$$9. \int \frac{x^3 - 3x^2 + 2}{x^2} dx = \int \left[\frac{x^3}{x^2} - \frac{3x^2}{x^2} + \frac{2}{x^2} \right] dx = \int \left[x - \frac{3}{x} + \frac{2}{x^2} \right] dx = \frac{x^2}{2} - 3x - \frac{2}{x} + C$$

$$10. \int 3 \csc x \cot x dx = 3 \int \csc x \cot x dx = 3(-\csc x) + C = -3 \csc x + C$$

$$11. \int 3 \sec x (2 \sec x + \tan x) dx = \int [6 \sec^2 x + 3 \sec x \tan x] dx = 6 \tan x + 3 \sec x + C$$

$$12. \int \frac{\csc x}{\sin x} dx = \int \frac{1/\sin x}{\sin x} dx = \int \frac{1}{\sin^2 x} dx = \int \csc^2 x dx = -\cot x + C$$

$$13. \int \frac{\sin \theta}{\cos^2 \theta} d\theta = \int \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} d\theta = \int \tan \theta \sec \theta d\theta = \sec \theta + C$$

$$14. \int [1 + \sin^2 \theta \csc \theta] d\theta = \int [1 + \sin^2 \theta \cdot \frac{1}{\sin \theta}] d\theta = \theta - \cos \theta + C$$

$$15. \int \tan^2 \beta d\beta = \int [\sec^2 \beta - 1] d\beta = \tan \beta - \beta + C$$

$$16. \text{ Find } y \text{ as a function of } x \text{ given } \frac{dy}{dx} = \sqrt[3]{x} \text{ and } y(1) = 2.$$

$$y(x) = \int x^{1/3} dx = \frac{3}{4} x^{4/3} + C$$

$$y(1) = \frac{3}{4} 1^{4/3} + C = 2 \Rightarrow C = 2 - \frac{3}{4} = \frac{5}{4}$$

$$\therefore y(x) = \frac{3}{4} x^{4/3} + \frac{5}{4}$$

$$17. \text{ Find } x \text{ as a function of } t \text{ given } \frac{dx}{dt} = \sin t + 1 \text{ and } x\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$x(t) = \int (\sin t + 1) dt = -\cos t + t + C$$

$$x(\pi/3) = -\cos \frac{\pi}{3} + \frac{\pi}{3} + C = \frac{1}{2}$$

$$-\frac{1}{2} + \frac{\pi}{3} + C = \frac{1}{2} \Rightarrow C = 1 - \frac{\pi}{3}$$

$$\therefore x(t) = -\cos t + t + 1 - \frac{\pi}{3}$$

$$18. \text{ Find } y \text{ as a function of } s \text{ given } \frac{dy}{ds} = \frac{s+1}{\sqrt{s}} \text{ and } y(1) = 0$$

$$y(s) = \int \frac{s+1}{\sqrt{s}} ds = \int (s^{1/2} + s^{-1/2}) ds = \frac{2}{3} s^{3/2} + 2s^{1/2} + C$$

$$y(1) = \frac{2}{3} (1)^{3/2} + 2(1)^{1/2} + C = 0 \Rightarrow C = -\frac{2}{3} - 2 = -\frac{8}{3}$$

$$\therefore y(s) = \frac{2}{3} s^{3/2} + 2s^{1/2} - \frac{8}{3}$$

$$19. \text{ Find the equation of the curve that passes through the point } (-3, 0) \text{ and whose slope at every}$$

$$\text{point } (x, y) \text{ is } 2x+1. \Rightarrow \frac{dy}{dx} = 2x+1 \Rightarrow y = \int (2x+1) dx = x^2 + x + C$$

$$y(-3) = 0 = (-3)^2 - 3 + C$$

$$0 = 9 - 3 + C$$

$$C = -6$$

$$\therefore \text{ the curve is the parabola } y = x^2 + x - 6$$