

Homework Set 18: Sigma Notation

Expand and evaluate:

$$1. \sum_{k=1}^3 k^3 = 1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36$$

$$2. \sum_{i=3}^5 2i+3 = (2(3)+3) + (2(4)+3) + (2(5)+3) = 9 + 11 + 13 = 33$$

$$3. \sum_{j=0}^5 j = 0 + 1 + 2 + 3 + 4 + 5 = 15$$

$$4. \sum_{i=1}^5 (-1)^i \cdot i = (-1)^1 \cdot 1 + (-1)^2 \cdot 2 + (-1)^3 \cdot 3 + (-1)^4 \cdot 4 + (-1)^5 \cdot 5 = -1 + 2 - 3 + 4 - 5 = -3$$

$$5. \sum_{k=1}^3 \cos k\pi = \cos(1 \cdot \pi) + \cos(2 \cdot \pi) + \cos(3 \cdot \pi) = -1 + 1 - 1 = -1$$

Express using sigma notation:

$$6. 1+2+3+4+\dots+9+10 = \sum_{k=1}^{10} k$$

$$7. 2+4+6+\dots+20 = \sum_{k=1}^{10} 2k$$

$$8. 3+5+7+\dots+17 = \sum_{k=1}^8 (2k+1)$$

$$9. 1-2+3-4+5 = \sum_{k=1}^5 (-1)^{k+1} \cdot k$$

$$10. 1-3+5-7+9-11+13-15 = \sum_{k=0}^7 (-1)^k (2k+1)$$

$$11. 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} = \sum_{k=1}^5 (-1)^{k+1} \cdot \frac{1}{k}$$

Evaluate the following sums:

$$12. \sum_{k=1}^{1000} k = \frac{1000(1000+1)}{2} = 500,500$$

$$13. \sum_{k=250}^{1000} k \begin{cases} \rightarrow (i) \sum_{k=1}^{1000} k - \sum_{k=1}^{249} k = 500,500 - \frac{249(249+1)}{2} = 469,375 \\ \text{OR} \\ \rightarrow (ii) \sum_{k=1}^{751} k + 249 = \sum_{k=1}^{751} k + \sum_{k=1}^{751} 249 = \frac{751(752)}{2} + 751(249) = 469,375 \end{cases}$$

$$14. \sum_{k=1}^{10} (5k+2) = 5 \sum_{k=1}^{10} k + \sum_{k=1}^{10} 2 = 5 \cdot \frac{10(10+1)}{2} + 10(2) = 275 + 20 = 295$$

$$15. \sum_{k=1}^{15} 3k^2 - k + 4 = 3 \sum_{k=1}^{15} k^2 - \sum_{k=1}^{15} k + \sum_{k=1}^{15} 4 = 3 \cdot \frac{15(15+1)(2 \cdot 15+1)}{6} - \frac{15(15+1)}{2} + 15 \cdot 4 = \frac{1240}{2} - \frac{120}{2} + \frac{60}{2} = 1180$$

$$16. \sum_{k=1}^{100} k(k+2)(k-1) = \sum_{k=1}^{100} (k^3 + k^2 - 2k) = \sum_{k=1}^{100} k^3 + \sum_{k=1}^{100} k^2 - 2 \sum_{k=1}^{100} k = \left[\frac{100(101)}{2} \right]^2 + \frac{100(101)201}{6} - 2 \cdot \frac{100(101)}{2} = 25,502,500 + 338,350 - 10,100 = 25,830,750$$

17. Express the sum of the even integers from 2 to 1000 in sigma notation, and then evaluate your sigma expression.

$$\sum_{k=1}^{500} 2k = 2 \sum_{k=1}^{500} k = 2 \cdot \frac{500(501)}{2} = 250,500$$