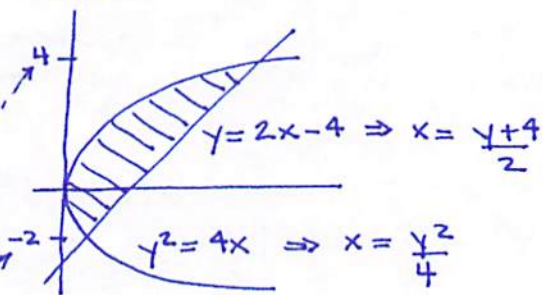


Homework Set 20 solutions

1) Sketch:

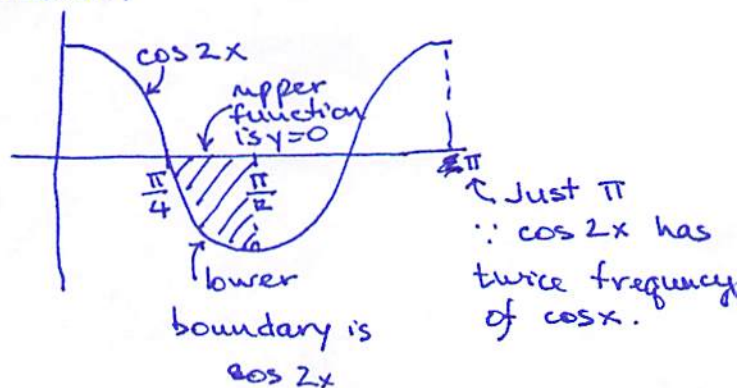


Points of intersection:

$$\begin{aligned} y^2 &= y^2 \\ 4x &= (2x-4)^2 \\ 4x &= 4x^2 - 16x + 16 \\ 0 &= 4x^2 - 20x + 16 \\ 0 &= x^2 - 5x + 4 \\ 0 &= (x-4)(x-1) \\ x &= 4 \quad x = 1 \\ y &= 4 \quad y = -2 \end{aligned}$$

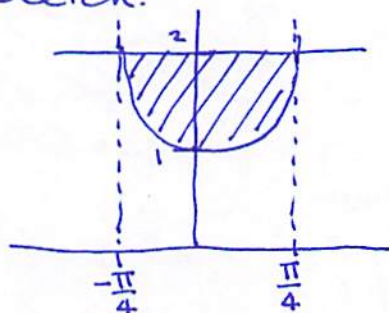
$$\begin{aligned} A &= \int_{-2}^4 \left[\frac{y+4}{2} - \frac{y^2}{4} \right] dy \\ &= \int_{-2}^4 \left[\frac{y}{2} + 2 - \frac{y^2}{4} \right] dy \\ &= \left[\frac{1}{2} \cdot \frac{y^2}{2} + 2y - \frac{1}{4} \cdot \frac{y^3}{3} \right]_{-2}^4 \\ &= \left[\frac{y^2}{4} + 2y - \frac{y^3}{12} \right]_{-2}^4 \\ &= \left(\cancel{4} + 8 - \frac{64}{12} \right) - \left(\cancel{1} - \cancel{4} + \frac{8}{12} \right) \\ &= 15 - \frac{72}{12} \\ &= 15 - 6 \\ &= 9 \end{aligned}$$

2) Sketch:



$$\begin{aligned} A &= \int_{\pi/4}^{3\pi/4} (0 - \cos 2x) dx \\ &= - \int_{\pi/4}^{3\pi/4} \cos 2x dx \\ &= - \left(+\frac{1}{2} \sin 2x \right) \Big|_{\pi/4}^{3\pi/4} \\ &= -\frac{1}{2} \left[\sin\left(2 \cdot \frac{3\pi}{4}\right) - \sin\left(2 \cdot \frac{\pi}{4}\right) \right] \\ &= -\frac{1}{2} \left[0 - 1 \right] \\ &= \frac{1}{2} \end{aligned}$$

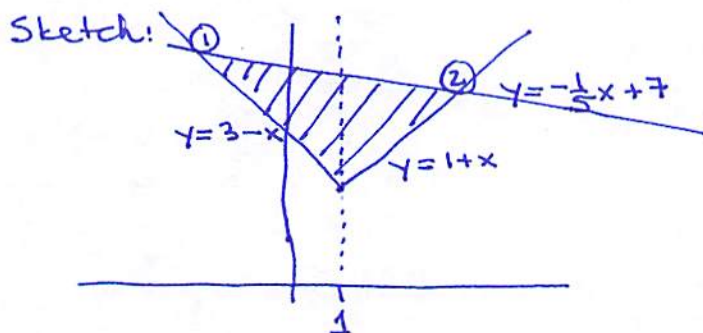
3) Sketch:



$$\begin{aligned}
 A &= \int_{-\pi/4}^{\pi/4} (2 - \sec^2 x) dx \\
 &= 2x - \tan x \Big|_{-\pi/4}^{\pi/4} \\
 &= (2 \cdot \frac{\pi}{4} - 1) - (2 \cdot (-\frac{\pi}{4}) - (-1)) \\
 &= \frac{\pi}{2} - 1 + \frac{\pi}{2} - 1 \\
 &= \pi - 2
 \end{aligned}$$

$$\begin{aligned}
 4) \quad y &= 2 + |x-1| = 2 + -(x-1), \quad x < 1 \\
 &= 3 - x
 \end{aligned}$$

$$\begin{aligned}
 &= 2 + (x-1), \quad x \geq 1 \\
 &= 1 + x
 \end{aligned}$$



Points of intersection:

① $3 - x = -\frac{1}{5}x + 7$

$$-\frac{4}{5}x = 4 \Rightarrow x = -5, \quad y = 3 - (-5) = 8$$

② $1 + x = -\frac{1}{5}x + 7$

$$\frac{6}{5}x = 6 \Rightarrow x = 5, \quad y = 1 + 5 = 6$$

$$\begin{aligned}
 A &= \int_{-5}^1 \left(-\frac{1}{5}x + 7\right) - (3 - x) dx \\
 &\quad + \int_1^5 \left(-\frac{1}{5}x + 7\right) - (1 + x) dx \\
 &= \int_{-5}^1 \frac{4}{5}x + 4 dx + \int_1^5 6 - \frac{6}{5}x dx \\
 &= \left[\frac{2x^2}{5} + 4x\right]_{-5}^1 + \left[6x - \frac{3x^2}{5}\right]_1^5 \\
 &= \left(\frac{2}{5} + 4\right) - \left(\frac{50}{5} - 20\right) + \left(30 - \frac{75}{5}\right) - \left(6 - \frac{3}{5}\right) \\
 &= \frac{72}{5} + \frac{48}{5} \\
 &= 24
 \end{aligned}$$

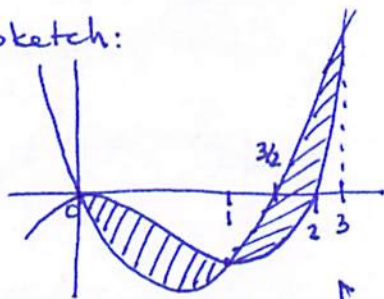
$$5) y = x^3 - 2x^2 = x^2(x-2)$$

$$x=0, x=2$$

$$y = 2x^2 - 3x = x(2x-3)$$

$$x=0, x=3/2$$

Sketch:



Intersections:

$$x^3 - 2x^2 \stackrel{!}{=} 2x^2 - 3x$$

$$x^3 - 4x^2 + 3x = 0$$

$$x(x^2 - 4x + 3) = 0$$

$$x(x-3)(x-1) = 0$$

$$x=0, 3, 1$$

$$A = \int_0^1 [(x^3 - 2x^2) - (2x^2 - 3x)] dx + \int_1^3 [(2x^2 - 3x) - (x^3 - 2x^2)] dx$$

$$= \int_0^1 [x^3 - 4x^2 + 3x] dx + \int_1^3 [-x^3 + 4x^2 - 3x] dx$$

$$= \left[\frac{x^4}{4} - \frac{4}{3}x^3 + \frac{3}{2}x^2 \right]_0^1 + \left[-\frac{x^4}{4} + \frac{4}{3}x^3 - \frac{3}{2}x^2 \right]_1^3$$

$$= \left(\frac{1}{4} - \frac{4}{3} + \frac{3}{2} \right) - 0 + \left(-\frac{3^4}{4} + \frac{4 \cdot 3^3}{3} - \frac{3 \cdot 3^2}{2} \right)$$

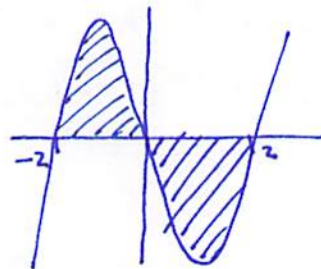
$$- \left(-\frac{1}{4} + \frac{4}{3} - \frac{3}{2} \right)$$

$$= \frac{2(3 - 16 + 18) - 3^5 + 4 \cdot 3^2 - 3 \cdot 3^2 \cdot 6}{12}$$

$$= \frac{37}{12}$$

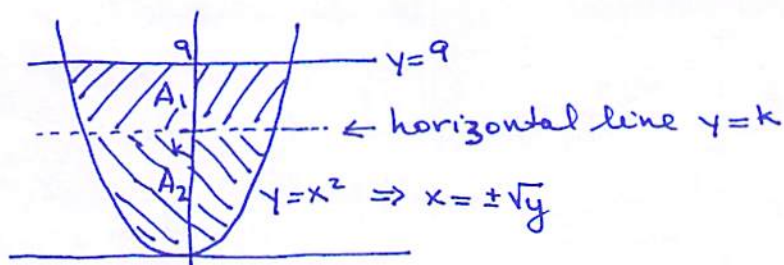
$$6) y = x^3 - 4x = x(x^2 - 4) = x(x+2)(x-2)$$

Sketch:



$$\begin{aligned} A &= 2 \int_0^2 (4x - x^3) dx \\ &= 2 \left[2x^2 - \frac{x^4}{4} \right]_0^2 \\ &= 2 \left[\left(2 \cdot 2^2 - \frac{2^4}{4} \right) - (0) \right] \\ &= 2 \left[8 - \frac{16}{4} \right] \\ &= 2[4] \\ &= 8 \end{aligned}$$

7) Sketch:



Find k such that $A_1 = A_2$

Solu Integrating with respect to y :

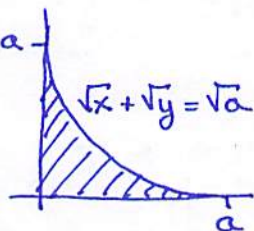
$$A_1 = \int_k^9 \sqrt{y} \, dy = A_2 = \int_0^k \sqrt{y} \, dy$$

$$\left[\frac{2}{3} y^{3/2} \right]_k^9 = \left[\frac{2}{3} y^{3/2} \right]_0^k$$

$$\begin{aligned} \frac{2}{3} 9^{3/2} - \frac{2}{3} k^{3/2} &= \frac{2}{3} k^{3/2} - 0 \\ \frac{2}{3} 9^{3/2} &= \frac{2}{3} k^{3/2} \\ \frac{27}{2} &= k^{3/2} \end{aligned}$$

$$\begin{aligned} \therefore k &= \left(\frac{27}{2} \right)^{2/3} \\ &= 9/\sqrt[3]{4} \end{aligned}$$

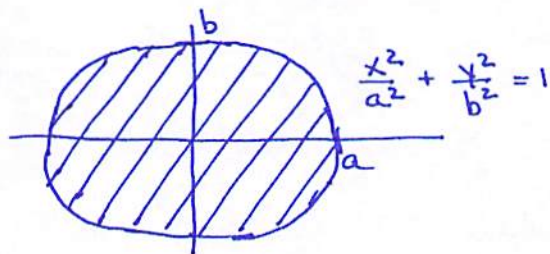
8) Sketch: a-



$$\begin{aligned} \sqrt{x} + \sqrt{y} &= \sqrt{a} \Rightarrow \sqrt{y} = \sqrt{a} - \sqrt{x} \\ y &= (\sqrt{a} - \sqrt{x})^2 \\ &= a - 2\sqrt{a}\sqrt{x} + x \end{aligned}$$

$$\begin{aligned} A &= \int_0^a (a - 2\sqrt{a}\sqrt{x} + x) \, dx \\ &= \left[ax - 2\sqrt{a} \frac{2}{3} x^{3/2} + \frac{x^2}{2} \right]_0^a \\ &= \left(a^2 - \frac{4}{3} \sqrt{a} a^{3/2} + \frac{a^2}{2} \right) - (0) \\ &= a^2 - \frac{4}{3} a^2 + \frac{a^2}{2} \\ &= \frac{6a^2 - 8a^2 + 3a^2}{6} \\ &= \frac{a^2}{6} \end{aligned}$$

a) Sketch:



Solve for y : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$y^2 = b^2 \left(1 - \frac{x^2}{a^2}\right)$$

$$y = \pm \sqrt{b^2 \left(1 - \frac{x^2}{a^2}\right)}$$

$$= \pm \sqrt{\frac{b^2}{a^2} (a^2 - x^2)}$$

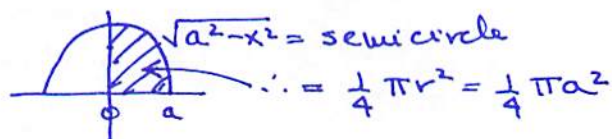
$$= \pm \frac{b}{a} \sqrt{a^2 - x^2}$$

+ ← top half
- ← bottom half

$$A = 4 \int_0^a \frac{b}{a} \sqrt{a^2 - x^2} \, dx$$

← Find area of $\frac{1}{4}$ of ellipse and multiply it by 4

$$= \frac{4b}{a} \left[\int_0^a \sqrt{a^2 - x^2} \, dx \right]$$



$$= \frac{4b}{a} \cdot \frac{1}{4} \pi a^2$$

$$= \pi ab.$$