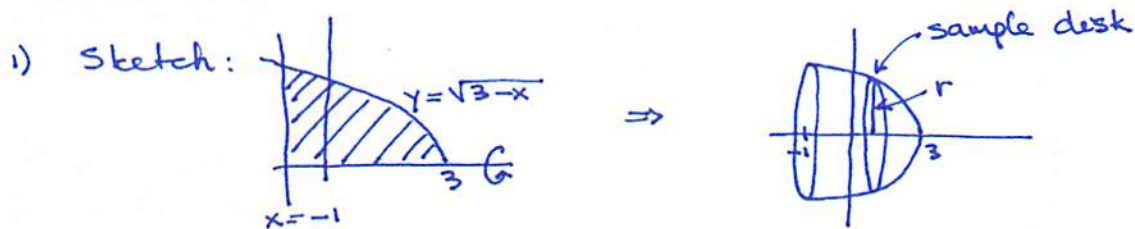
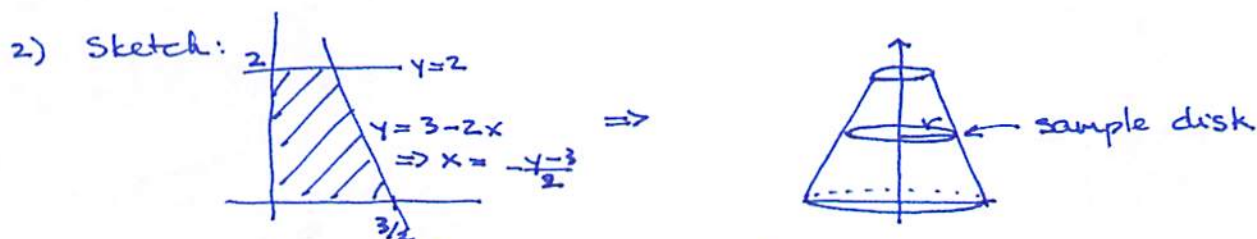


Homework Set 21: Solutions



$$V = \pi \int r^2 dx = \pi \int_{-1}^3 (\sqrt{3-x})^2 dx = \pi \int_{-1}^3 (3-x) dx = \pi \left[3x - \frac{x^2}{2} \right]_{-1}^3$$

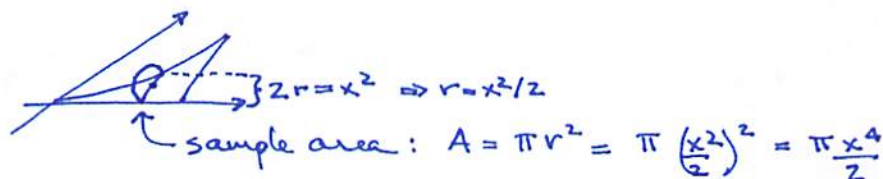
$$= \pi \left[(9 - 9/2) - (-3 - 1/2) \right] = \pi \left[9/2 + 7/2 \right] = 8\pi$$



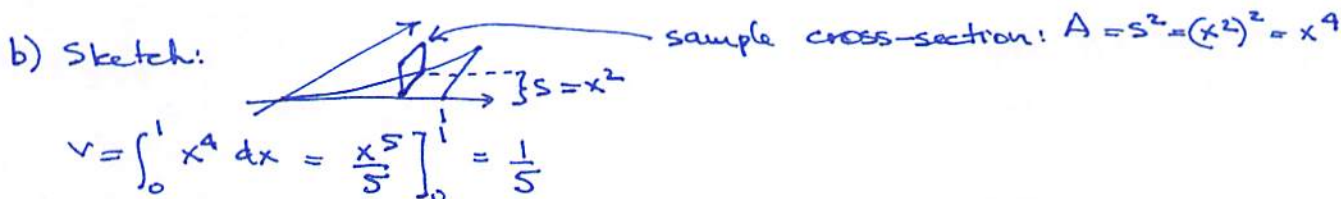
$$V = \pi \int r^2 dy = \pi \int_0^2 \left(\frac{3-y}{2} \right)^2 dy = \frac{\pi}{4} \int_0^2 (9 - 6y + y^2) dy$$

$$= \frac{\pi}{4} \left[9y - 3y^2 + \frac{y^3}{3} \right]_0^2 = \frac{\pi}{4} \left[(18 - 12 + \frac{8}{3}) - (0) \right] = \frac{\pi}{4} \cdot \frac{14}{3} = \frac{13\pi}{6}$$

3) a) Sketch:

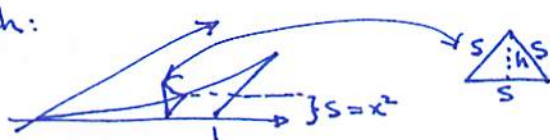


$$V = \int_0^1 \frac{\pi x^4}{2} dx = \frac{\pi}{2} \left[\frac{x^5}{5} \right]_0^1 = \pi/10$$



$$V = \int_0^1 x^4 dx = \left[\frac{x^5}{5} \right]_0^1 = \frac{1}{5}$$

c) Sketch:

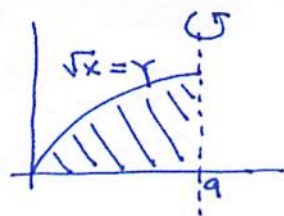


$$h^2 + \left(\frac{s}{2} \right)^2 = s^2 \quad h = \sqrt{s^2 - \frac{s^2}{4}} = \frac{\sqrt{3}}{2} s$$

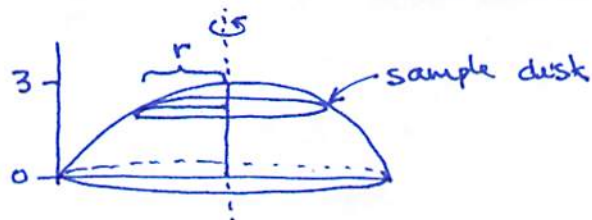
$$A(\Delta) = \frac{1}{2} b h = \frac{1}{2} s \cdot \frac{\sqrt{3}}{2} s = \frac{\sqrt{3}}{4} s^2$$

$$V = \int_0^1 \frac{\sqrt{3}}{4} s^2 dx = \int_0^1 \frac{\sqrt{3}}{4} (x^2)^2 dx = \frac{\sqrt{3}}{4} \left[\frac{x^5}{5} \right]_0^1 = \frac{\sqrt{3}}{20}$$

4) Sketch:



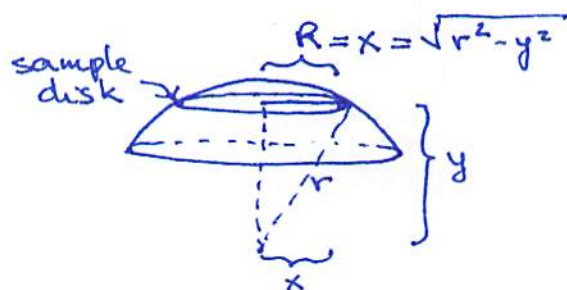
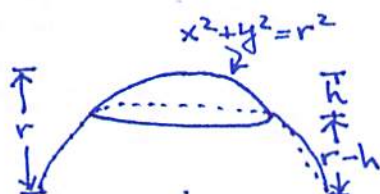
⇒



$$V = \pi \int_0^3 r^2 dy = \pi \int_0^3 (9 - y^2)^2 dy \quad \because \sqrt{x} = y \Rightarrow x = y^2 + \text{shift} \Rightarrow 9 - y^2$$

$$= \pi \int_0^3 (81 - 18y^2 + y^4) dy = \pi \left[81y - 6y^3 + \frac{y^5}{5} \right] = \pi \left[243 - 6(27) + \frac{3^5}{5} \right] = \frac{648\pi}{5}$$

5) Sketch:



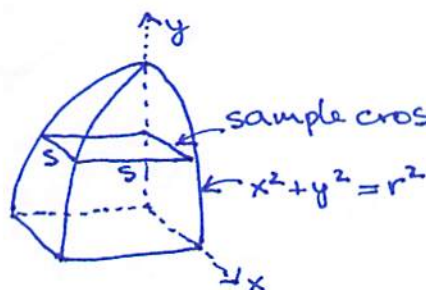
$$V = \pi \int_{r-h}^r (\sqrt{r^2 - y^2})^2 dy = \pi \int_{r-h}^r (r^2 - y^2) dy = \pi \left[r^2 y - \frac{y^3}{3} \right]_{r-h}^r$$

$$= \pi \left[\left(r^3 - \frac{r^3}{3} \right) - \left(r^2(r-h) - \frac{(r-h)^3}{3} \right) \right]$$

$$= \pi \left[\frac{2}{3} r^3 - r^3 + r^2 h + \frac{1}{3} (r^3 - 3r^2 h + 3rh^2 - h^3) \right]$$

$$= \pi \left[\left(\frac{2}{3} - 1 + \frac{1}{3} \right) r^3 + (1-1) r^2 h + rh^2 - \frac{h^3}{3} \right] = \pi h^2 (r - h/3) = \pi h^2 (3r - h)/3$$

6) Sketch:



$$= x^2$$

$$= r^2 - y^2$$

$$V = 8 \int_0^r (r^2 - y^2) dy = 8 \left[r^2 y - \frac{y^3}{3} \right]_0^r = 8 \left[\left(r^3 - \frac{r^3}{3} \right) - (0) \right] = 8 \cdot \frac{2}{3} r^3 = \frac{16r^3}{3}$$