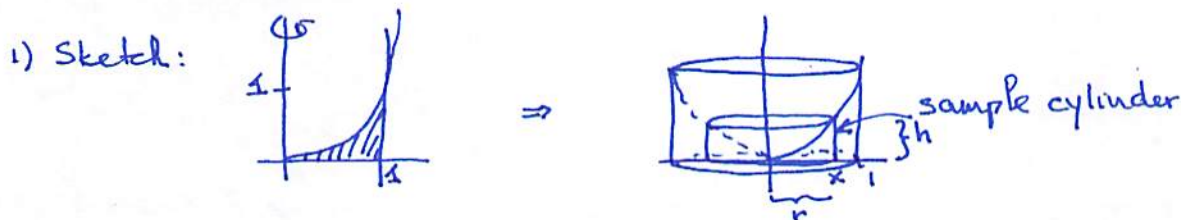
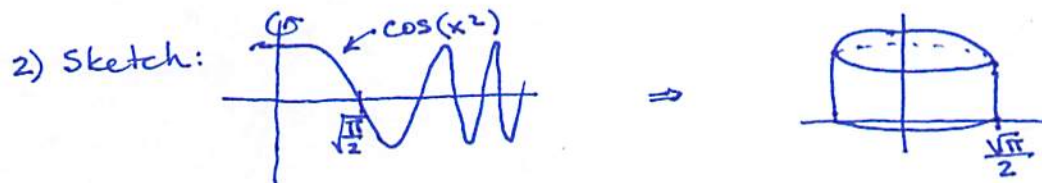


Homework set 22: Solutions

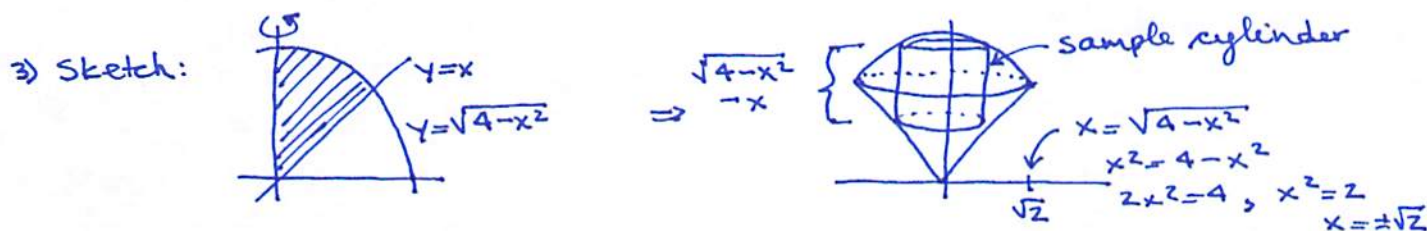


$$V = 2\pi \int_0^1 r h dx = 2\pi \int_0^1 x \cdot x^3 dx = 2\pi \int_0^1 x^4 dx = \left[\frac{2\pi}{5} x^5 \right]_0^1 = \frac{2\pi}{5}$$



$$V = 2\pi \int_0^{\sqrt{\pi}/2} x \cos(x^2) dx \quad \text{via } u = x^2, \frac{du}{2} = x dx, \quad x=0 \Rightarrow u=0^2=0, \quad x=\sqrt{\pi}/2 \Rightarrow u=(\sqrt{\pi}/2)^2 = \pi/4$$

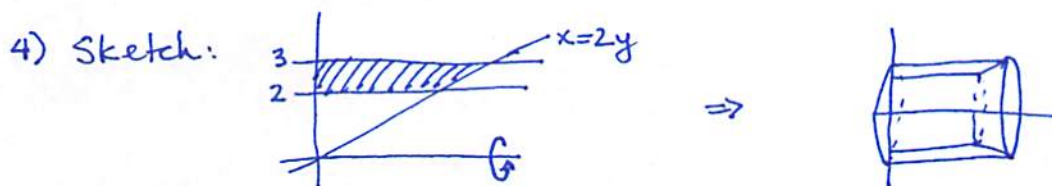
$$= \pi \int_0^{\pi/4} \cos u du = \pi [\sin u]_0^{\pi/4} = \pi [\frac{\sqrt{2}}{2} - 0] = \frac{\pi\sqrt{2}}{2}$$



$$V = 2\pi \int_0^{\sqrt{2}} x (\sqrt{4-x^2} - x^2) dx = 2\pi \int_0^{\sqrt{2}} (x\sqrt{4-x^2} - x^3) dx$$

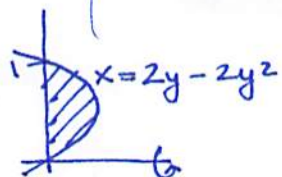
$$= 2\pi \left[\frac{2}{3} (4-x^2)^{3/2} \left(-\frac{1}{2}\right) - \frac{x^4}{4} \right]_0^{\sqrt{2}} = \frac{2\pi}{3} \left[-(4-x^2)^{3/2} - x^4 \right]_0^{\sqrt{2}}$$

$$= \frac{2\pi}{3} \left[-(4-2)^{3/2} - (\sqrt{2})^4 - (-(4-0)^{3/2} - 0^4) \right] = \frac{2\pi}{3} [8 - 2\sqrt{8}] = \frac{8\pi}{3} (2 - \sqrt{2})$$

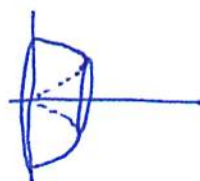


$$V = 2\pi \int_2^3 y(2y) dy = 4\pi \int_2^3 y^2 dy = \frac{4\pi}{3} [y^3]_2^3 = \frac{4\pi}{3} [27 - 8] = \frac{76\pi}{3}$$

5) Sketch:

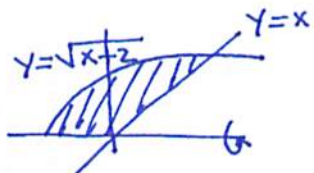


$\Rightarrow$

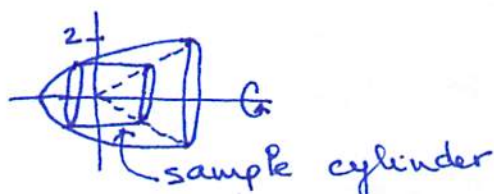


$$V = 2\pi \int_0^1 y(2y - 2y^2) dy = 4\pi \int_0^1 (y^2 - y^3) dy = 4\pi \left[\frac{y^3}{3} - \frac{y^4}{4} \right]_0^1 = 4\pi \left[\frac{1}{3} - \frac{1}{4} \right] = \frac{\pi}{3}$$

6) Sketch:



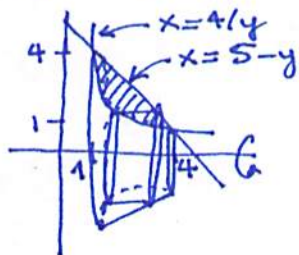
$\Rightarrow$



$$V = 2\pi \int_0^2 y(y - (y^2 - 2)) dy = 2\pi \int_0^2 (y^2 - y^3 + 2y) dy$$

$$= 2\pi \left[\frac{y^3}{3} - \frac{y^4}{4} + y^2 \right]_0^2 = 2\pi \left[\frac{8}{3} - \frac{16}{4} + 4 \right] = 2\pi \left[\frac{8}{3} \right] = \frac{16\pi}{3}$$

7) Sketch:

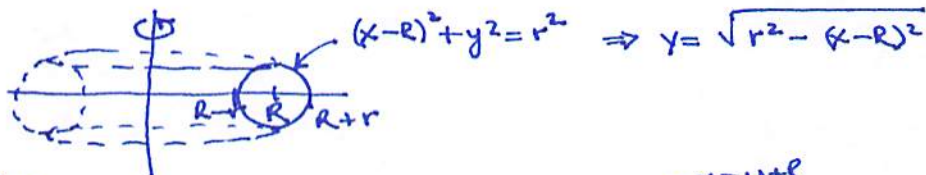


$$V = 2\pi \int_1^4 y(5 - y - \frac{4}{y}) dy = 2\pi \int_1^4 (5y - y^2 - 4) dy$$

$$= 2\pi \left[\frac{5y^2}{2} - \frac{y^3}{3} - 4y \right]_1^4 = 2\pi \left[\left(\frac{80}{2} - \frac{64}{3} - 16 \right) - \left(\frac{5}{2} - \frac{1}{3} - 4 \right) \right]$$

$$= 9\pi$$

8) Sketch:



$$V = 2\pi \int_{R-r}^{R+r} x \sqrt{r^2 - (x-R)^2} dx$$

let $u = x - R \Rightarrow x = u + R$
 $du = dx$
 $x = R - r \Rightarrow u = R - r - R = -r$
 $x = R + r \Rightarrow u = R + r - R = r$

$$= 2\pi \int_{-r}^r (u + R) \sqrt{r^2 - u^2} du$$

$$= 2\pi \int_{-r}^r \left(u \sqrt{r^2 - u^2} + R \sqrt{r^2 - u^2} \right) du$$

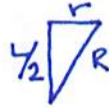
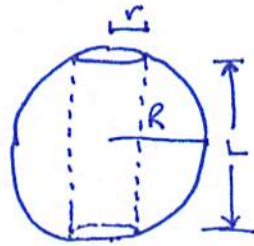
let $v = u^2$
 $\frac{dv}{2} = u du$
 $u = r \Rightarrow v = r^2$
 $u = -r \Rightarrow v = r^2$

$$= \frac{2\pi}{2} \int_{r^2}^{r^2} \frac{\sqrt{r^2 - v}}{2} dv + 2\pi R \int_{-r}^r \sqrt{r^2 - u^2} du$$

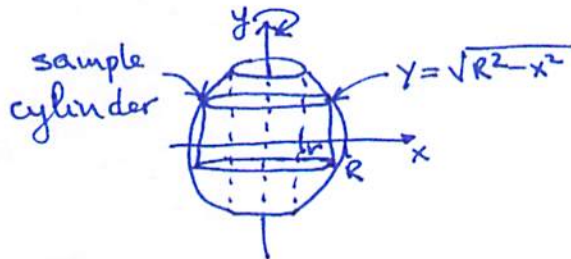
The first term is zero. The second term is the area of a semicircle with radius r, which is $\frac{\pi r^2}{2}$.

$$= \frac{2}{2} \pi R \cdot \frac{\pi r^2}{2} = 2\pi^2 R r^2$$

a) Sketch:



$$r^2 + \left(\frac{L}{2}\right)^2 = R^2$$



$$\begin{aligned} V &= 2\pi \int_r^R 2x \sqrt{R^2 - x^2} dx = 2\pi \left[\frac{2}{3} (R^2 - x^2)^{3/2} \left(-\frac{1}{x}\right) \right]_r^R = -\frac{4\pi}{3} \left[\frac{(R^2 - R^2)^{3/2}}{(R^2 - r^2)^{3/2}} \right] \\ &= -\frac{4\pi}{3} \left[-\left(\frac{L}{2}\right)^3 \right] \\ &= \frac{4\pi}{3} \left(\frac{L}{2}\right)^3 \\ &= \frac{\pi L^3}{6} \end{aligned}$$

Since this expression contains L but not R the volume must not depend on R .