

## Homework Set 24: Solutions

1)  $y = 7x$ ,  $0 \leq x \leq 1$ ,  $\curvearrowright$  x-axis  $f'(x) = 7 \Rightarrow [f'(x)]^2 = 49$ .

$$S = 2\pi \int_0^1 7x \sqrt{1+49} dx = 2\pi \cdot 7 \cdot \sqrt{50} \int_0^1 x dx = \frac{70\pi\sqrt{2}}{2} x^2 \Big|_0^1 = 35\pi\sqrt{2}$$

2)  $y = \sqrt{4-x^2}$ ,  $-1 \leq x \leq 1$ ,  $\curvearrowright$  x-axis  $(S = 2\pi \int f(x) \sqrt{1+[f'(x)]^2} dx)$

$$f'(x) = \frac{1}{x\sqrt{4-x^2}} \cdot -2x = \frac{-x}{\sqrt{4-x^2}} \Rightarrow [f'(x)]^2 = \frac{x^2}{4-x^2}$$

$$S = 2\pi \int_{-1}^1 \sqrt{4-x^2} \sqrt{1 + \frac{x^2}{4-x^2}} dx \rightarrow 1 + \frac{x^2}{4-x^2} = \frac{4-x^2+x^2}{4-x^2} = \frac{4}{4-x^2}$$

$$= 2\pi \int_{-1}^1 \sqrt{4-x^2} \sqrt{\frac{4}{4-x^2}} dx = 2\pi \int_{-1}^1 \sqrt{4-x^2} \frac{2}{\sqrt{4-x^2}} dx = 4\pi \int_{-1}^1 dx$$

$$= 4\pi [x]_{-1}^1 = 4\pi [(1) - (-1)] = 8\pi$$

3)  $y = \sqrt{x} - \frac{1}{3}x^{3/2}$ ,  $1 \leq x \leq 3$ ,  $\curvearrowright$  x-axis

$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{\sqrt{x}}{2} \Rightarrow 1 + [f'(x)]^2 = 1 + \frac{1}{4x} - \frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{2} + \frac{x}{4} = \frac{1}{4x} + \frac{1}{2} + \frac{x}{4} = \left(\frac{1}{2\sqrt{x}} + \frac{\sqrt{x}}{2}\right)^2$$

$$S = 2\pi \int_1^3 \left(\sqrt{x} - \frac{x^{3/2}}{3}\right) \sqrt{\left(\frac{1}{2\sqrt{x}} + \frac{\sqrt{x}}{2}\right)^2} dx$$

$$= 2\pi \int_1^3 \left(\frac{1}{2} + \frac{x}{2} - \frac{x}{6} - \frac{x^2}{6}\right) dx = \frac{\pi}{3} \int_1^3 (3 + 2x - x^2) dx$$

$$= \frac{\pi}{3} \left[ 3x + x^2 - \frac{x^3}{3} \right]_1^3 = \frac{\pi}{3} \left[ (9+9-9) - (3+1-\frac{11}{3}) \right] = \frac{16\pi}{9}$$

4)  $x = 9y+1$ ,  $0 \leq y \leq 2$ ,  $\curvearrowright$  y-axis  $\Rightarrow S = 2\pi \int g(y) \sqrt{1+[g'(y)]^2} dy$

$$g(y) = 9y+1 \Rightarrow g'(y) = 9 \Rightarrow S = 2\pi \int_0^2 (9y+1) \sqrt{1+81} dy = 2\pi\sqrt{82} \int_0^2 (9y+1) dy$$

$$= 2\pi\sqrt{82} \left[ \frac{9}{2}y^2 + y \right]_0^2 = 2\pi\sqrt{82} [(18+2) - 0] = 40\pi\sqrt{82}$$

$$5) \quad x = \sqrt{9-y^2}, \quad -2 \leq y \leq 2, \quad \text{Oy-axis}$$

$$g'(y) = \frac{-2y}{2\sqrt{9-y^2}} = \frac{-y}{\sqrt{9-y^2}} \Rightarrow 1 + [g'(y)]^2 = 1 + \left(\frac{-y}{\sqrt{9-y^2}}\right)^2 = 1 + \frac{y^2}{9-y^2} = \frac{9-y^2+y^2}{9-y^2} = \frac{9}{9-y^2}$$

$$S = 2\pi \int_{-2}^2 \sqrt{9-y^2} \sqrt{\frac{9}{9-y^2}} dy = 6\pi \int_{-2}^2 dy = 6\pi [y]_{-2}^2 = 6\pi [(2) - (-2)] = 24\pi.$$

$$6) \quad 8xy^2 = 2y^6 + 1, \quad 1 \leq y \leq 2, \quad \text{Oy-axis}$$

$$x = \frac{2y^6}{8y^2} + \frac{1}{8y^2} = \frac{y^4}{4} + \frac{1}{8y^2} \Rightarrow g'(y) = y^3 - \frac{1}{4y^3}$$

$$\Rightarrow 1 + [g'(y)]^2 = 1 + \left(y^3 - \frac{1}{4y^3}\right)^2 = 1 + y^6 - \frac{1}{2} + \frac{1}{16y^6} = \left(y^3 + \frac{1}{4y^3}\right)^2$$

$$S = 2\pi \int_1^2 \left(\frac{y^4}{4} + \frac{1}{8y^2}\right) \sqrt{\left(y^3 + \frac{1}{4y^3}\right)^2} dy$$

$$= 2\pi \int_1^2 \left(\frac{y^7}{4} + \frac{\frac{3y}{16}}{\frac{y^4}{4}} + \frac{1}{32y^5}\right) dy = 2\pi \frac{1}{32} \int_1^2 \left(8y^7 + 6y + \frac{1}{y^5}\right) dy$$

$$= \frac{\pi}{16} \left[ y^8 + 3y^2 - \frac{1}{4y^4} \right]_1^2 = \frac{\pi}{16} \left[ \left(2^8 + 12 - \frac{1}{64}\right) - \left(1 + 3 - \frac{1}{4}\right) \right]$$

$$= \frac{16,911\pi}{1024}$$