

Homework set 16: Probability

Useful facts about a deck of cards: 4 suits (clubs, diamonds, hearts and spades) each in 13 denominations (Ace, 2, 3, ..., Queen, King) for a total of 52 cards.

1. A card is to be dealt from a shuffled deck.

- What is the probability that it will be the Ace of Clubs? $\frac{1}{52} \approx 0.0192$
- What is the probability that it will be a Club? $\frac{1}{4} = 0.25$ or $\frac{13}{52} = 0.25$
- What is the probability that it will be a face card? $(J, Q, K) \times 4 = 12$ so $\frac{12}{52} \approx 0.231$
- What is the probability that it will be a face card and a club? $\frac{3}{52} \approx 0.058$
- What is the probability that it will be a face card or a club? $13 \text{ clubs} + 9 \text{ other face cards} = 22 \Rightarrow \frac{22}{52} \approx 0.423$
- What is the probability of dealing some card? $\frac{52}{52} = 1.0$
- What is the probability of dealing a hockey card? $\frac{0}{52} = 0.0$

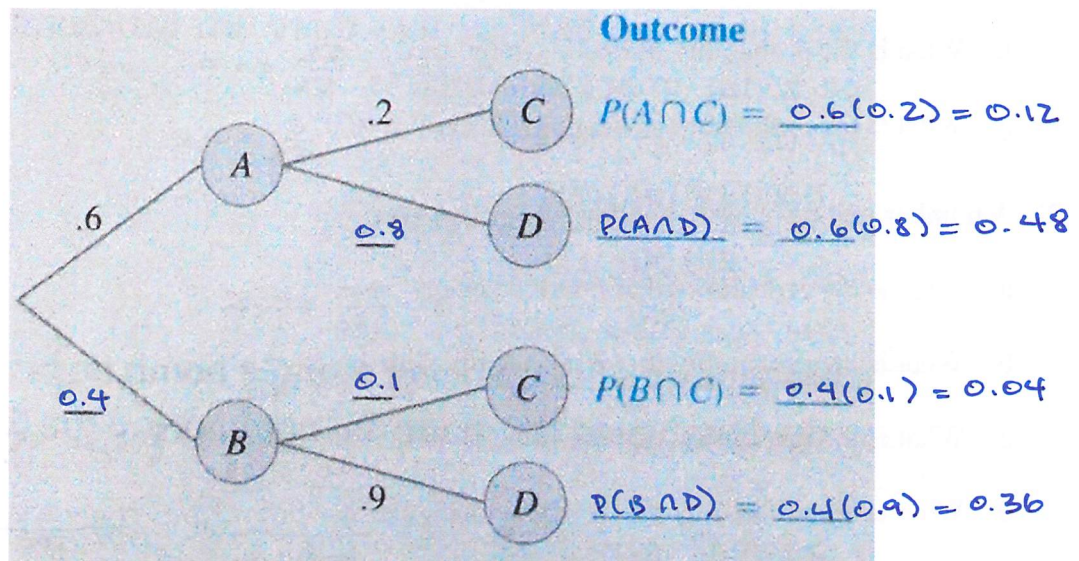
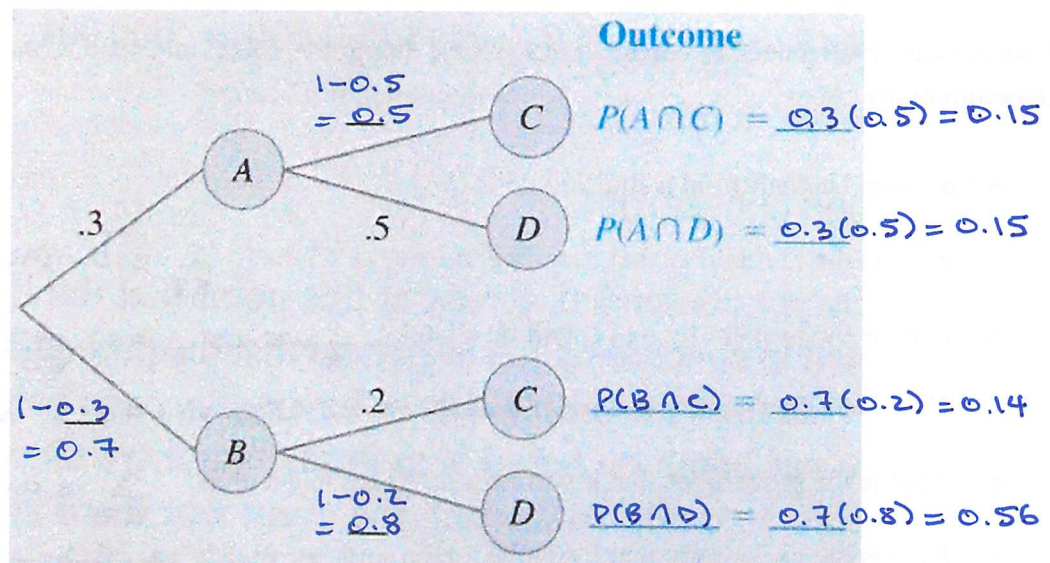
2. A number will be chosen randomly from 1 to 100.

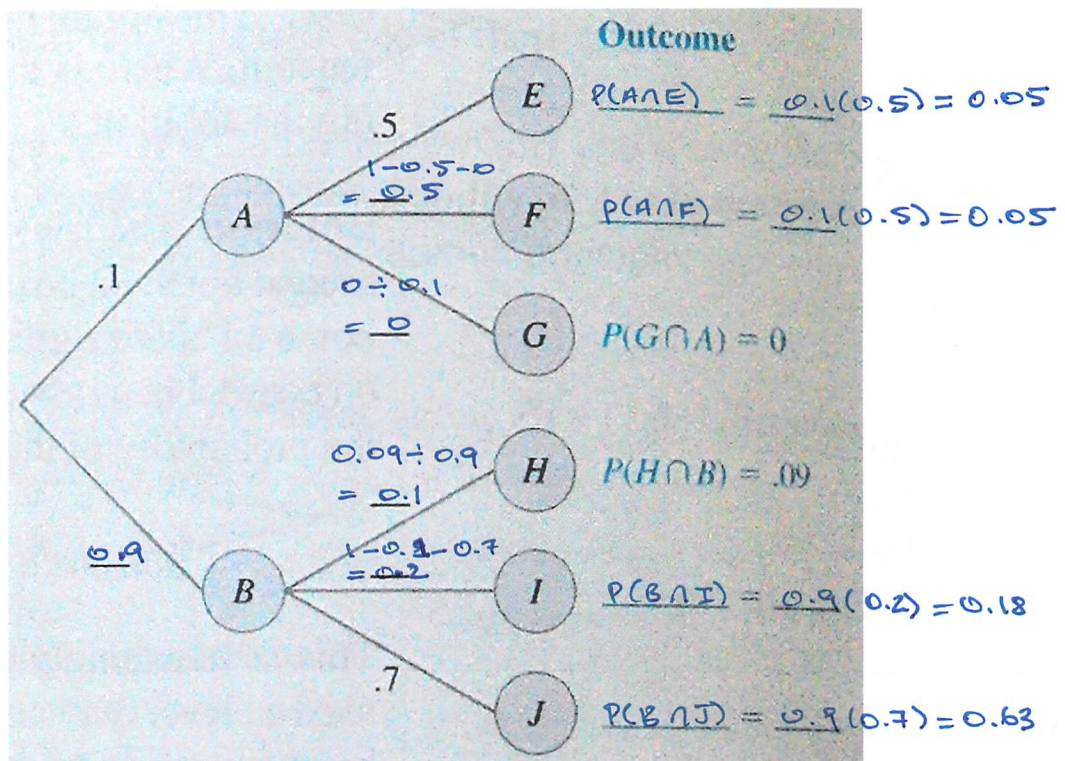
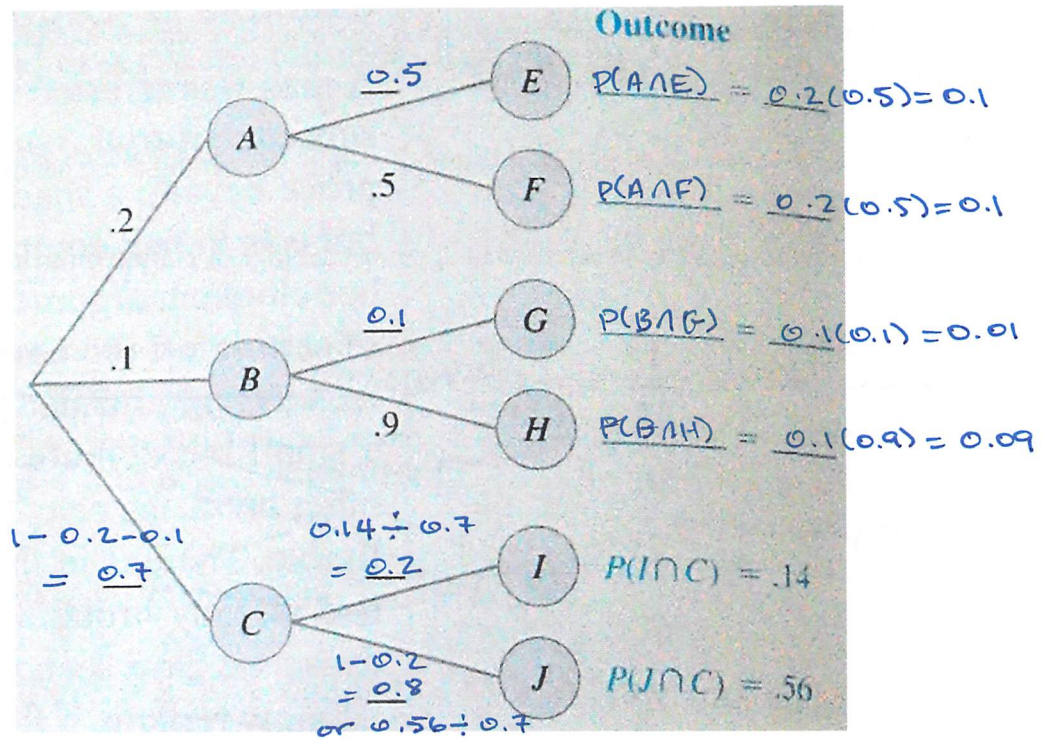
- What is the probability that it will be even? $\frac{50}{100} = 0.5$
- What is the probability that it will be divisible by 5? $\frac{20}{100} = 0.2$
- What is the probability that it will be divisible by 5 and by 3? $\{15, 30, 45, 60, 75, 90\} \Rightarrow \frac{6}{100} = 0.06$
- What is the probability that it will be divisible by 5 or by 3? $\frac{20 + 33 - 6}{100} = \frac{47}{100} = 0.47$
both: 6
- What is the probability that it will be divisible by 5 but not by 3? $\frac{20 - 6}{100} = \frac{14}{100} = 0.14$
- What is the probability that it will be between 30 and 60 (inclusive), and odd, but not divisible by 3? $\frac{10}{100} = 0.1$
 $31 \neq 3 \quad \div 2 = 15$
 $-\{33, 39, 45, 51, 57\} = 10$

~~30~~	31	~~32~~	~~33~~	~~34~~	35	~~36~~	37	~~38~~	39
40	41	~~42~~	43	~~44~~	~~45~~	46	47	~~48~~	49
~~50~~	~~51~~	~~52~~	53	~~54~~	55	~~56~~	~~57~~	~~58~~	59
~~60~~									

$/$ = even ones
 $/$ = divisible by 3
■ = final set

3. Supply the missing values in each of the tree diagrams:





4. Three ordinary quarters and a fake quarter with two heads are placed in a hat. One coin is selected at random and tossed three times.

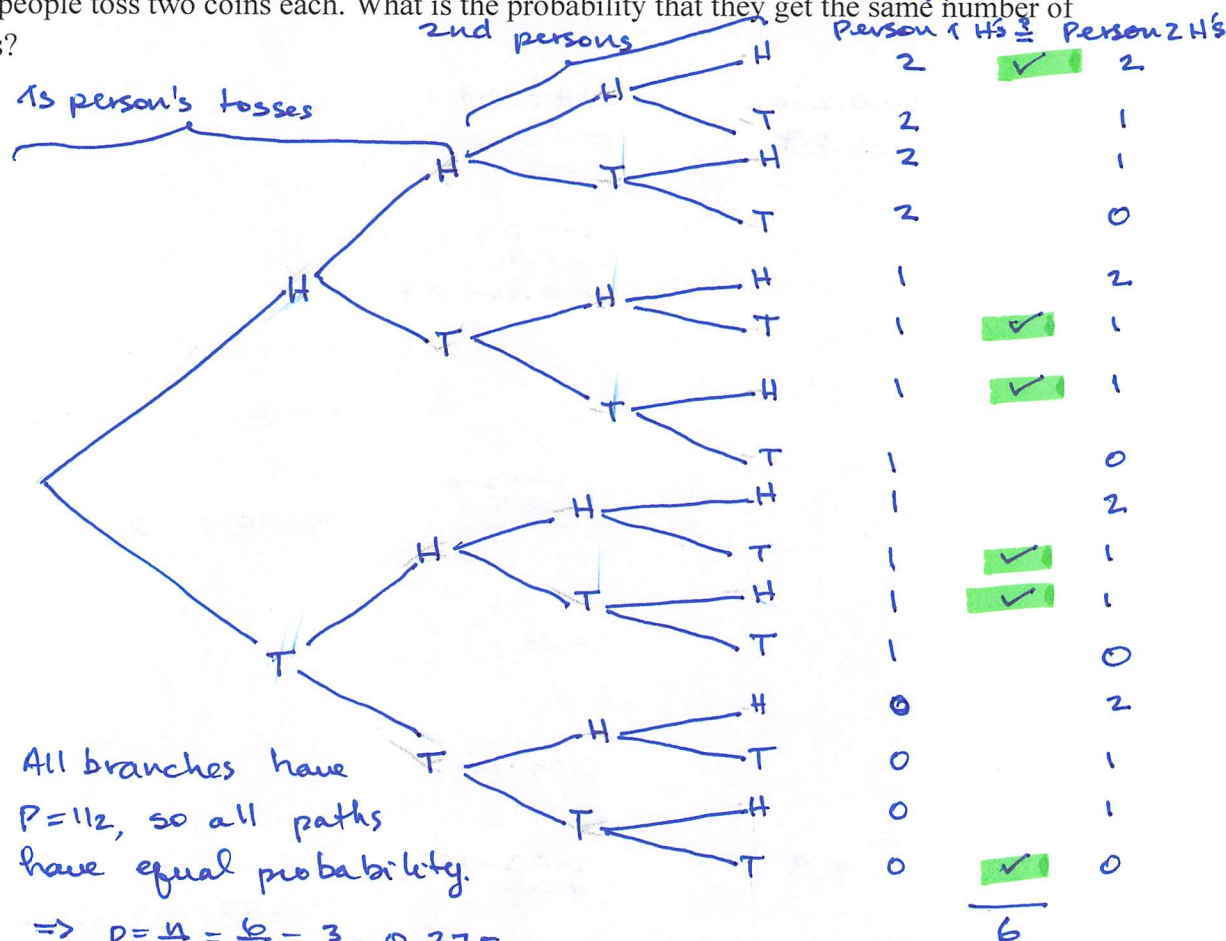
a. If the fake quarter is selected what is the probability of getting three heads? 1.0

b. If an ordinary quarter is selected what is the probability of getting three heads? $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

c. If the result is three heads, what is the probability that the fake quarter was selected?

From diagram below: $p(\text{Fake} | \text{HHH}) = \frac{p(\text{Fake} \cap \text{HHH})}{p(\text{Fake} \cap \text{HHH}) + p(\text{Real} \cap \text{HHH})} = \frac{1/4}{1/4 + 3/32} = 0.72$

5. Two people toss two coins each. What is the probability that they get the same number of heads?



4c)

