

Combinatorics Formulas

The idea How do we count how many possibilities there are without counting each one individually (enumerating them)?

Multiplication principle If some event can occur in m different ways, and following that a second event can occur in n ways, then the number of ways the two events can occur in order is,

$$m \times n$$

Factorial Number of ways of ordering n items,

$$n! = n \times (n - 1) \times (n - 2) \times (n - 3) \times \cdots \times 2 \times 1$$

(Note: $0! = 1$)

Permutations (lineups and partial lineups): The number of ways of ordering r items chosen from n items.

$$P(n, r) = {}_nP_r = \frac{n!}{(n - r)!}$$

Combinations (committees and subsets): The number of possible subsets of r items chosen from among n items,

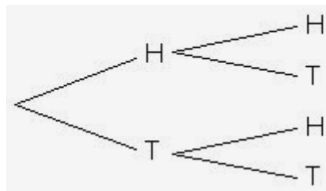
$$C(n, r) = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n - r)!}$$

Specific case Number of ways a sequence of n binary events (e.g. coin tosses) can play out,

$$2 \times 2 \times 2 \times 2 \times \cdots \times 2 = 2^n$$

Models

Tree



Fill in the blanks

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Bernoulli trials

1. Two outcomes per trial: success or failure
2. Constant probability of success
3. Outcome of each trial is independent of any other trial

Binomial Distribution

The distribution of the number of successes in n Bernoulli trials.

Individual probability of k successes in n Bernoulli trials,

$$p(k|n) = \binom{n}{k} p^k (1 - p)^{n-k}$$