

1. (10 marks) *Bird Nests*. In a study of bird nesting habits Martin, Aitken and Wiebe¹ reported the following observations about their study plot:
 - It contained 10,719 available trees, 690 of which contained at least one nest.
 - It contained four tree species: 1,562 Quaking aspen, 3,260 Douglas-fir, 4,296 Lodgepole pine, and 1,601 Hybrid spruce.
 - There were nests in 658 of the Quaking aspen, 9 of the Douglas-fir, 18 of the Lodgepole pine, and 6 of the Hybrid spruce.
 - 1,612 nests were observed in Quaking aspen trees, 19 in Douglas-fir, 48 in Lodgepole pine and 10 in Hybrid spruce.
 - In one subsample of trees with nests 87 trees had a single nest, 51 of which were in use that season, 99 trees had 2 or 3 nests, in 33 of which at least one nest was in use, and 33 trees had more than 3 nests, only 5 of which had nests in use that season.
 - a. If a tree is selected at random what is the probability that it is a Quaking aspen? That it is a Douglas-fir?
 - b. If a tree is selected at random what is the probability that it has at least one nest in it?
 - c. If a tree is a Quaking aspen what is the probability that it has at least one nest in it? What if it is a Douglas-fir?
 - d. If a tree has a nest in it what is the probability that it is a Quaking aspen? That it is a Douglas-fir?
 - e. If you spot a nest in a tree what is the probability that it is the only nest in the tree?
 - f. If you spot a nest in a tree what is the probability that it has been in use this season?
2. (5 marks) *Smelts*. A recent survey of mitochondrial DNA variation in smelts in a lake revealed that two haplotypes (genotypes) were present. Forty percent of individual fish in the population were haplotype A. The remaining 60 percent were haplotype B. If an experimenter were to sample 10 fish from this population, what is the probability that none are haplotype A? At least 5 are haplotype B?

¹ Martin, K., K.E.H. Aitken and K.L. Wiebe. 2004. Nest sites and nest webs for cavity-nesting communities in interior British Columbia, Canada: nest characteristics and niche partitioning. *Condor* 106: 5-19

3. (10 marks) *Paradise flycatchers*. A population of paradise flycatchers has 80% brown males and 20% white. You capture five male flycatchers at random.
- What is the chance that at least two are white?
 - You need to band at least four white ones. How many should you plan to capture to have a 50% chance of getting four white ones?
 - Your supervisor wants you to be “almost certain” of banding four white ones. How many should you plan to capture to have a 90% chance of getting four white ones?
4. (10 marks) *About that probability*. Estimate the probability that in the mark-recapture problem done in “class” the value of N is between:
- 55 and 65 inclusive².
 - 50 and 70 inclusive.
5. (5 marks) *Pitcher problems*. You are at the side of a river. You have a three-litre pitcher and a seven-litre pitcher. The pitchers do not have markings to allow measuring smaller quantities—all you can do is fill them from the river, dump them into the river or pour from one to the other.
- You need four litres of water. How can you measure four litres?
 - You need two litres of water. Can you measure two litres?
6. (20 marks) *March Madness*. Each Spring³ the NCAA holds its championship basketball tournament. This is a knockout tournament (teams are eliminated after their first loss) in which 64 teams compete to determine the champion. A selection committee selects the teams to participate and *seeds* them in four regions (South, West, Midwest and East) according to their estimated strength from strongest (1) to weakest (16).

² *inclusive* means including the endpoints, i.e. including 55 and 65

³ Well until 2020 when COVID-19 forced cancellation of the tournament!

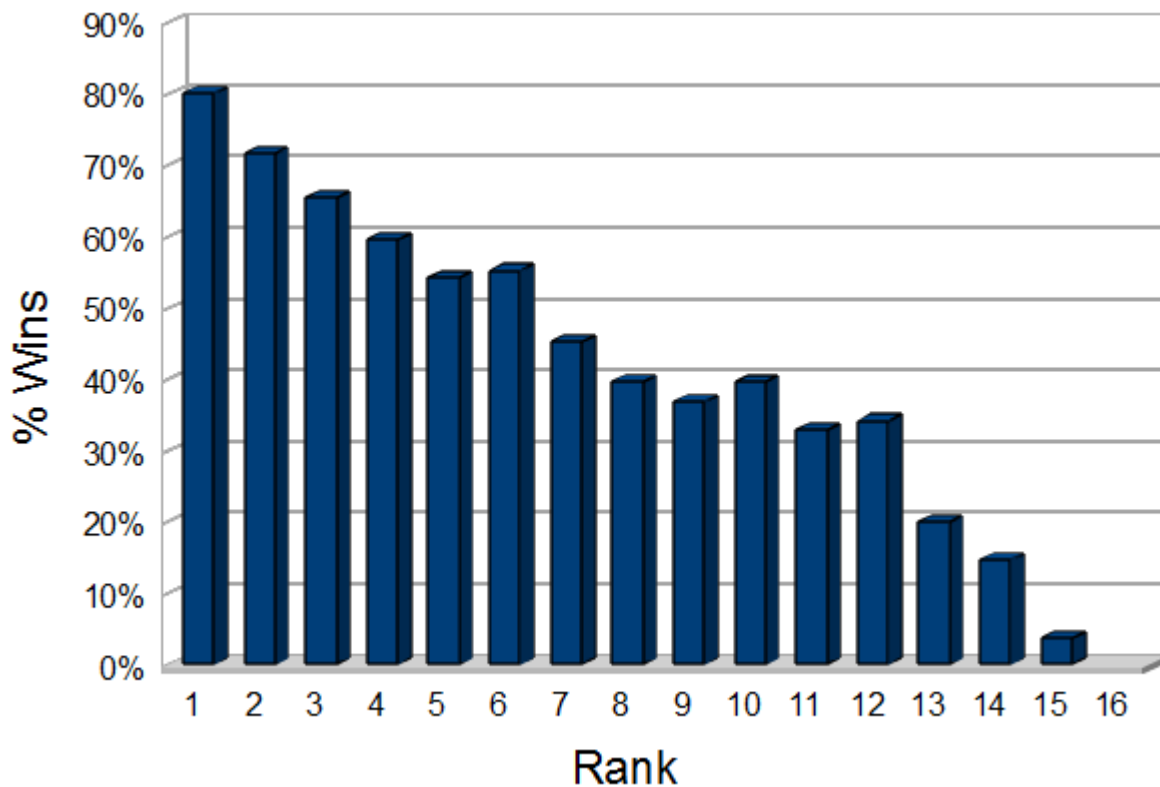


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Each year there are contests to try and predict the outcomes of the games, and in particular to pick the outcomes of *all* the games correctly. One year the contest that caught everyone's attention was the Quicken Loans Billion Dollar Bracket Challenge which offered to award \$1,000,000,000 (a billion dollars) to any person who could correctly pick the winners of **all** the games, and \$100,000 to the people with the 20 most accurate "imperfect" brackets.

- How many games will be played in the tournament?
- How many possible ways are there for the tournament to play out?
- If one just guessed randomly at the outcomes of each game what is the probability of getting them all correct?

The odds in c. above are poor. How can we do better? By making informed guesses rather than random ones! The outcomes of all games are not of course equally likely. For example as of 2018 no number 1 seed had ever (in 116 games!) lost to a number 16 seed. Here are the %s of their games teams have historically won according to their seeding rank.



- d. Using the data above what would you predict the outcomes of the games in the tournament to be?
- e. What is the probability that your prediction in d. is correct?
- f. You may have found that the data in the graph above were not helpful in predicting the outcomes of some games in d. More detailed data is available at <http://rotoguru2.com/hoop/mad/seedwl.html> Does it enable you to make a better prediction? If so, what is your improved prediction and how good do you think it is? If not, why not?
- g. If the contest were to be held next year do you think someone would win it? Why or why not?