

Minimal Spanning Tree Algorithm¹: On-the-graph

The Problem

Find the set of edges of minimum weight that spans the graph. To span a graph means to connect all the nodes in the graph.

(Note that the resulting set of edges will be a tree, i.e. a graph with no circuits, because if there is a circuit at least one of the edges in the circuit will be superfluous and thus can be removed to lighten the set.)

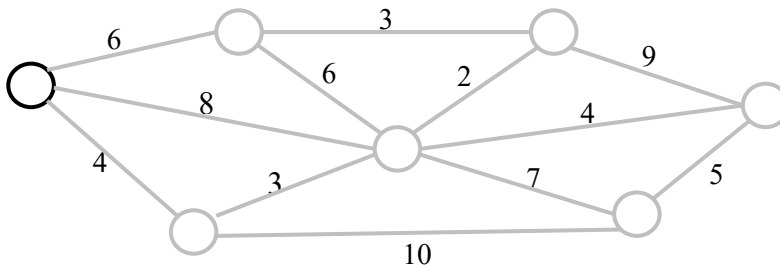
Algorithm

The MST algorithm builds the tree up by starting with a single node and then adding one edge to it on each iteration². It doesn't matter which node you begin with. The key is to add the edge to the closest unconnected node to the tree on each iteration.

1. Begin by selecting any node arbitrarily.
2. Circle it to show it is in the tree.
3. Identify the lightest edge that connects to a new (unconnected) node, and add this edge to the tree. Circle the newly connected node to show that is now also in the tree. Continue until all nodes have been added.

Example

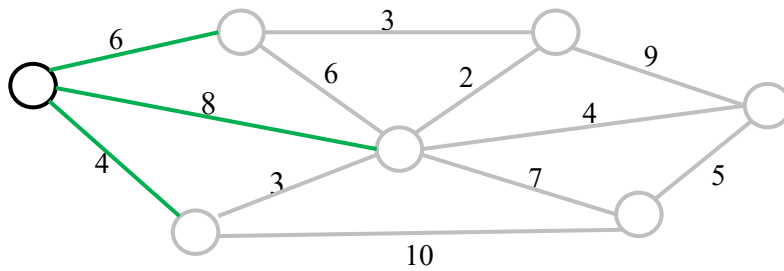
We begin by arbitrarily selecting the leftmost node and using it to create our initial tree.



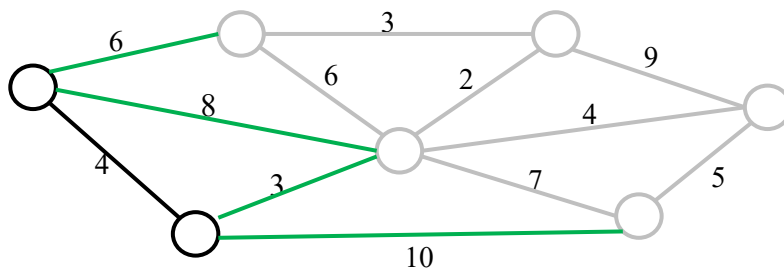
There are three edges to consider adding to the tree. These are shown in green.

¹ An algorithm is a step-by-step set of operations to be performed to solve a problem. (Fine print: the number of steps must be finite, they must be guaranteed to finish, to produce a correct answer, and to be mechanical, i.e. not to require any ingenuity.)

² “**Iteration** is the act of repeating a process, either to generate an unbounded sequence of outcomes, or with the aim of approaching a desired goal, target or result. Each repetition of the process is also called an “**iteration**”, and the results of one **iteration** are used as the starting point for the next **iteration**.” [https://en.wikipedia.org/wiki/Iteration]

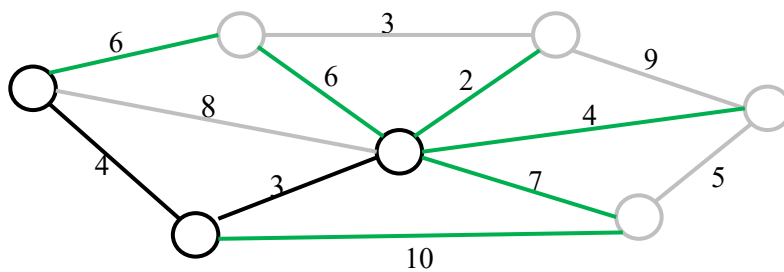


We will add the “lightest” one (weight 4).

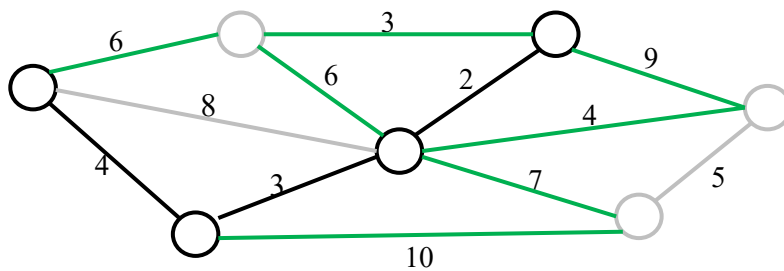


Adding that edge gives us access to two new edges (weights 3 and 10) to consider as well as the remaining two original ones (weights 6 and 8).

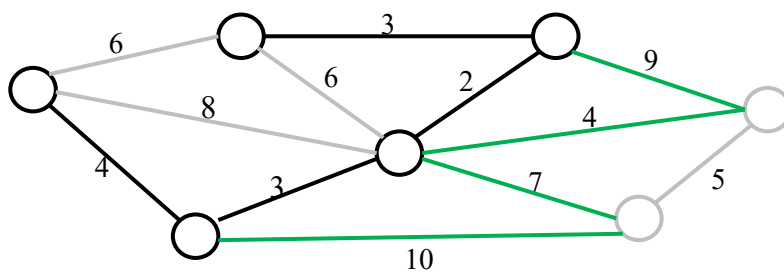
We again select the lightest one (weight 3) and add it to our growing tree.



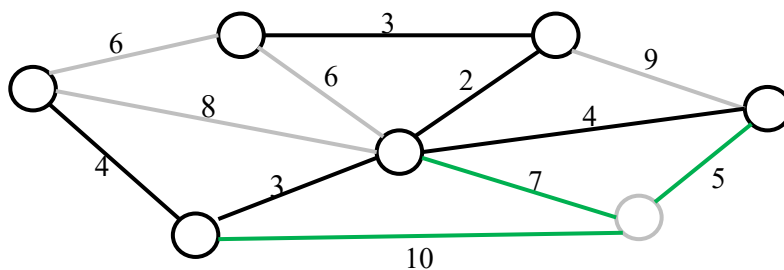
As before this gives us access to new potential edges (weights 6, 2, 4 and 7), but it also eliminates one edge from consideration (weight 8) because we do not add edges that link to already connected nodes (since those nodes are already in the tree). We add the lightest edge (weight 2).



This provides two new possibilities with weights 3 and 9 in addition to our earlier possibilities. The 3 is the lightest so we add it..



Adding the 3 eliminates the two 6s from consideration since they only connect us to nodes already in the tree. The 4 is the lightest edge to a “new” node so we add it to the tree.



Now we connect the final node using the lightest edge (weight 5). The edges that make up the minimal spanning tree are shown in black. The grey edges are ones that are not part of the minimal spanning tree.

