

Shortest Route Algorithm: In-the-matrix

The Problem

Find the path of least weight from the origin node to the destination node.

Algorithm

The shortest route algorithm fans out from the origin growing multiple paths. On each iteration it finds the shortest path to a “new” (so far unreached) node in the graph. It continues until it reaches the destination node.

Example

Find the shortest route from A to F in the graph below.

	A	B	C	D	E	F
A	—	7	4	12	14	—
B	7	—	2	8	7	11
C	4	2	—	7	5	10
D	12	8	7	—	1	6
E	14	7	5	1	—	9
F	—	11	10	6	9	—

A is our starting node so we “circle” its row label, and cross out its column.

	A	B	C	D	E	F
A	~~/~~	7	4	12	14	—
B	~~/~~	—	2	8	7	11
C	~~/~~	2	—	7	5	10
D	~~/~~	8	7	—	1	6
E	~~/~~	7	5	1	—	9
F	~~/~~	11	10	6	9	—

We look for the shortest route we can build from the node(s) we have reached, in this case only A. The possibilities are the edges in the A row of the matrix. The smallest value is 4, so we:

- Circle that element,
- Cross out the rest of its column,
- Circle the corresponding row label, i.e. C, and
- Add 4 to each entry in row C (to show the total cost of extending a path beyond C).

	A	B	C	D	E	F
A	~~/~~	7	4	12	14	–
B	~~/~~	–	~~2~~	8	7	11
+4 C	~~4~~	~~2~~ 6	~~/~~	~~7~~ 11	~~5~~ 9	~~10~~ 14
D	~~12~~	8	~~/~~	–	1	6
E	~~14~~	7	~~5~~	1	–	9
F	~~/~~	11	~~10~~	6	9	–

We have two vertices we can grow from now, the circled rows A and C. We look for the smallest number in either row and find the 6 in the C row, so we:

- Circle that element,
- Cross out the rest of its column,
- Circle the corresponding row label, i.e. B, and
- Add 6 to each entry in row B (to show the total cost of extending a path beyond B).

	A	B	C	D	E	F
A	~~/~~	~~/~~	4	12	14	–
+6 B	~~/~~	~~/~~	~~2~~	~~8~~ 14	~~7~~ 13	~~11~~ 17
+4 C	~~4~~	~~2~~ 6	~~/~~	~~7~~ 11	~~5~~ 9	~~10~~ 14
D	~~12~~	~~8~~	~~/~~	–	1	6
E	~~14~~	~~/~~	~~5~~	1	–	9
F	~~/~~	~~11~~	~~10~~	6	9	–

We look for the smallest entry in rows A, B and C, find the 9 in row C, so we:

- Circle that element,
- Cross out the rest of its column,
- Circle the corresponding row label, i.e. E, and
- Add 9 to each entry in row E (to show the total cost of extending a path beyond E).

	A	B	C	D	E	F
A	~~/~~	~~/~~	4	12	~~14~~	–
+6 B	~~/~~	~~/~~	~~2~~	~~8~~ 14	~~7~~ 13	~~11~~ 17
+4 C	~~4~~	~~2~~ 6	~~/~~	~~7~~ 11	~~5~~ 9	~~10~~ 14
D	~~12~~	~~8~~	~~/~~	–	~~1~~	6
+9 E	~~14~~	~~/~~	~~5~~	~~1~~ 10	~~/~~	~~9~~ 18
F	~~/~~	~~11~~	~~10~~	6	~~9~~	–

We look for the smallest entry in rows A, B, C and E, find the 10 in row D, so we:

- Circle that element,
- Cross out the rest of its column,
- Circle the corresponding row label, i.e. D, and
- Add 10 to each entry in row D (to show the total cost of extending a path beyond D).

	A	B	C	D	E	F
A	~~/~~	~~7~~	4	~~12~~	~~14~~	—
+6 B	~~7~~	~~/~~	~~2~~	~~8⁺⁶~~ ~~14~~	~~7⁺⁶~~ ~~13~~	~~11⁺⁶~~ ~~17~~
+4 C	~~4~~	~~2⁺⁴~~ 6	~~/~~	~~7⁺⁴~~ ~~11~~	~~5⁺⁴~~ 9	~~10⁺⁴~~ ~~14~~
+10 D	~~12~~	~~8~~	~~7~~	—	~~1~~	~~6⁺¹⁰~~ ~~16~~
+9 E	~~14~~	~~7~~	~~5~~	~~1⁺⁹~~ 10	~~/~~	~~9⁺⁹~~ ~~18~~
F	~~/~~	~~11~~	~~10~~	~~6~~	~~9~~	—

Finally we look for the smallest number in rows A through E, find the 14, so we:

- Circle that element,
- Cross out the rest of its column,
- Circle the corresponding row label, i.e. D, and show it took a path of length 14 to reach it.

	A	B	C	D	E	F
A	~~/~~	~~7~~	4	~~12~~	~~14~~	~~/~~
+6 B	~~7~~	~~/~~	~~2~~	~~8⁺⁶~~ ~~14~~	~~7⁺⁶~~ ~~13~~	~~11⁺⁶~~ ~~17~~
+4 C	~~4~~	~~2⁺⁴~~ 6	~~/~~	~~7⁺⁴~~ ~~11~~	~~5⁺⁴~~ 9	~~10⁺⁴~~ 14
+10 D	~~12~~	~~8~~	~~7~~	—	~~1~~	~~6⁺¹⁰~~ ~~16~~
+9 E	~~14~~	~~7~~	~~5~~	~~1⁺⁹~~ 10	~~/~~	~~9⁺⁹~~ ~~18~~
+14 F	~~/~~	~~11~~	~~10~~	~~6~~	~~9~~	—

Now we know the length of the shortest path from A to F is 14, but which edges make up the path? It's not necessarily all the “circled” edges in the graph, because some of those are along paths to other places.

To find out which ones make up the shortest path we work backwards from F to A:
 we got to F from C,
 we got to C from A,
 so the shortest path is A-C-F with weight 14.