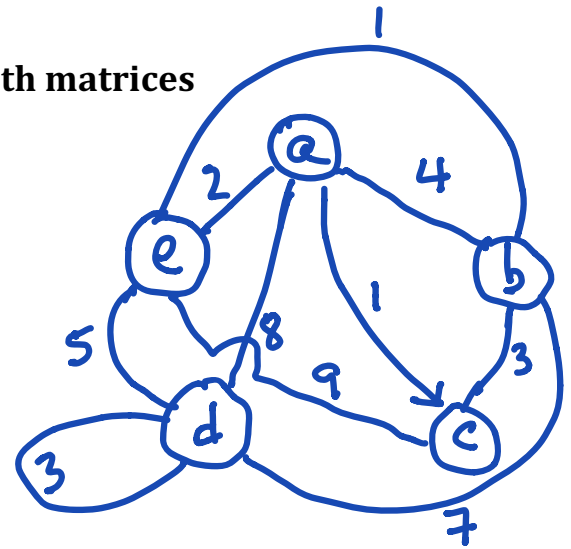


Exercise set 12.2: Graph theory with matrices

1. Consider the graph whose weighted adjacency matrix is:

	a	b	c	d	e
a		4	1	8	2
b	4		3	7	1
c		3			9
d	3	7			5
e	2	1	9	5	



a) Is it directed or undirected?

asymmetric $\therefore$ directed

b) How many vertices does it have?

5 = # rows = # cols

c) How many edges does it have?

10: 8 bidirectional, 1 directional $a \rightarrow c$, 1 loop at d

d) Are there any loops?

1 at d

e) What is its minimal spanning tree?

$a \begin{matrix} \swarrow c \\ \searrow e \end{matrix} \begin{matrix} \swarrow b \\ \searrow d \end{matrix}$ Total weight = 9

f) Draw a picture of it.

2. Find the minimal spanning tree for the graph whose weighted adjacency matrix is:

	a	b	c	d	e
a		3	6	9	12
b	3		5	7	4
c	6	5		5	6
d	9	7	5		2
e	12	4	6	2	

Edges: $a \begin{matrix} \swarrow c \\ \searrow b \end{matrix} \begin{matrix} \swarrow d \\ \searrow e \end{matrix}$ $b \begin{matrix} \swarrow e \\ \searrow c \end{matrix}$ $c \begin{matrix} \swarrow b \\ \searrow d \end{matrix}$ $d \begin{matrix} \swarrow c \\ \searrow e \end{matrix}$ $e \begin{matrix} \swarrow d \\ \searrow b \end{matrix}$

Total weight: 17

3. Find the minimal spanning tree for the graph whose weighted adjacency matrix is:

	a	b	c	d	e	f	g	h	i	j	k
a	-	12	8	11	14	5	7	9	13	15	6
b	12	-	7	12	15	8	3	5	8	11	4
c	8	7	-	8	5	12	15	13	7	4	9
d	11	12	8	-	8	13	11	16	12	8	6
e	14	15	5	8	-	7	7	12	9	14	12
f	5	8	12	13	7	-	9	0	11	8	7
g	7	3	15	11	7	9	-	2	10	6	7
h	9	5	13	16	12	10	12	-	14	12	11
i	13	8	7	12	9	11	10	4	-	10	8
j	15	11	4	8	14	8	6	2	10	-	6
k	6	4	9	6	12	7	7	11	8	6	-

Edges
af
ak
bg
bh
ce
ci
gj
jc
kb
kd

Weights
5
6
3
5
5
7
6
4
4
6

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4. What is the shortest route from a to k in the graph whose weighted adjacency matrix is:

	a	b	c	d	e	f	g	h	i	j	k
a	-	6	5	14	13	-	-	-	-	-	-
b	6	-	2	8	14	18	-	-	-	-	-
c	5	2	-	-	9	13	23	-	-	-	-
d	14	8	-	-	8	17	26	25	-	-	-
e	13	14	9	8	-	4	18	12	23	26	-
f	18	14	13	17	4	-	-	6	9	9	-
g	23	26	25	26	18	-	-	25	31	26	-
h	25	26	25	26	12	6	-	-	8	30	-
i	23	26	31	26	9	8	8	-	-	-	33
j	26	9	26	9	26	9	30	30	-	-	27
k	-	-	-	-	-	-	-	-	33	27	-

Total length

Route: a - b - d - g - j - k
6 8 9 3 1 = 27