

1. (10 marks) *Reachability*

Consider the adjacency matrix,

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>
<i>a</i>	1	0	1	0	0	1	0	0	1	0
<i>b</i>	0	0	0	1	0	0	1	0	0	0
<i>c</i>	1	0	0	0	0	1	0	1	1	0
<i>d</i>	0	1	0	1	0	0	1	0	0	0
<i>e</i>	0	0	0	0	0	1	0	1	1	1
<i>f</i>	1	0	1	0	1	0	0	0	1	1
<i>g</i>	0	1	0	1	0	0	1	0	0	0
<i>h</i>	0	0	1	0	1	0	0	0	1	0
<i>i</i>	1	0	1	0	1	1	0	1	0	1
<i>j</i>	0	0	0	0	1	1	0	0	1	0

- How many paths of length 2 are there from *f* to *i*?
- How many paths of length 4 are there from *a* to *e*?
- Determine if the graph is connected¹ by calculating as many adjacency matrices as necessary and combining them into a reachability matrix.

Remember: Excel is your friend! (I'm not expecting you to multiply this out by hand!)

2. (10 marks) *The rule of 72*

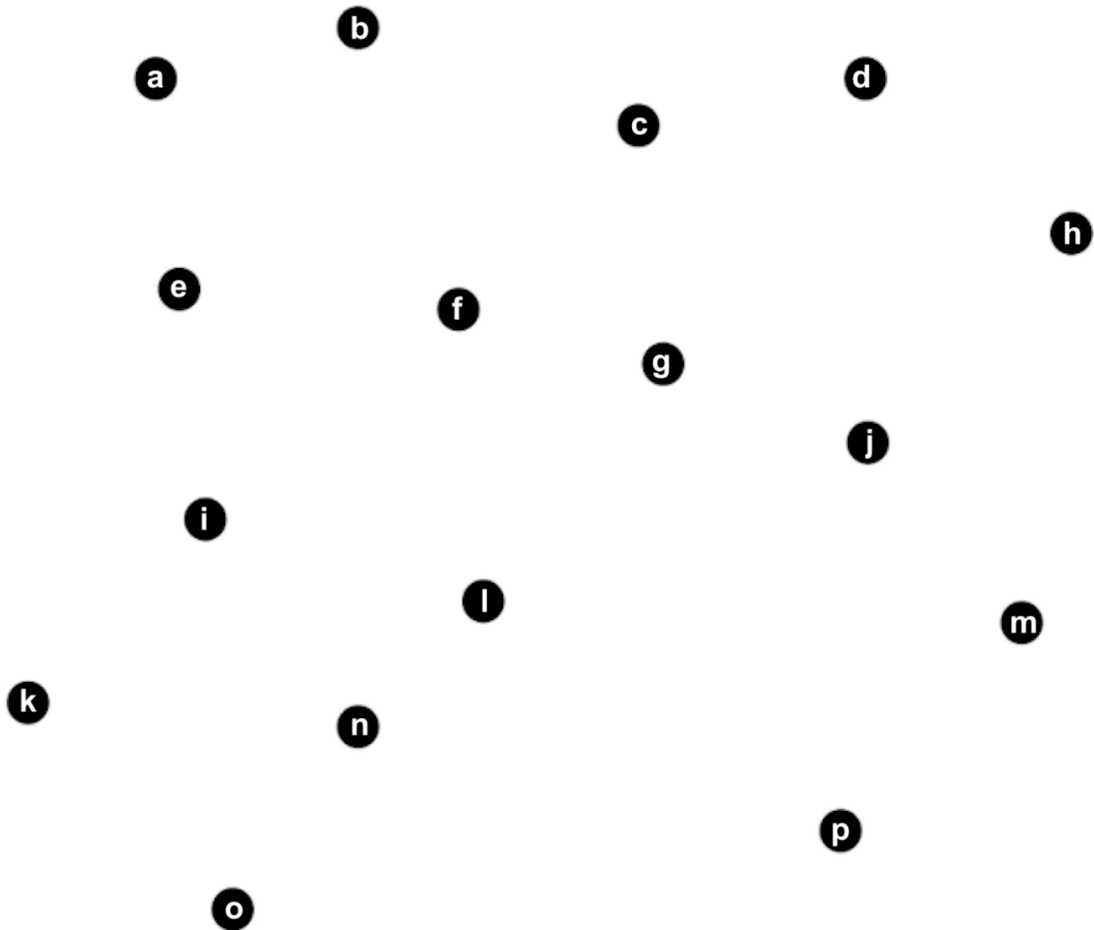
The rule of 72 is a very useful heuristic. But if we are going to rely on a heuristic we should know how good or bad the answers are going to be.

- Find the exact doubling time for interest rates of 1, 2, 3, ..., 20 percent.
- Find the doubling time given by the heuristic for each of these interest rates.
- What is the percentage error of the heuristic for each? If you told someone about this heuristic trick what would you tell them about the results?
- Derive a similar heuristic approximation for tripling time, that is the time it would take an investment to triple. Be sure to show your work.

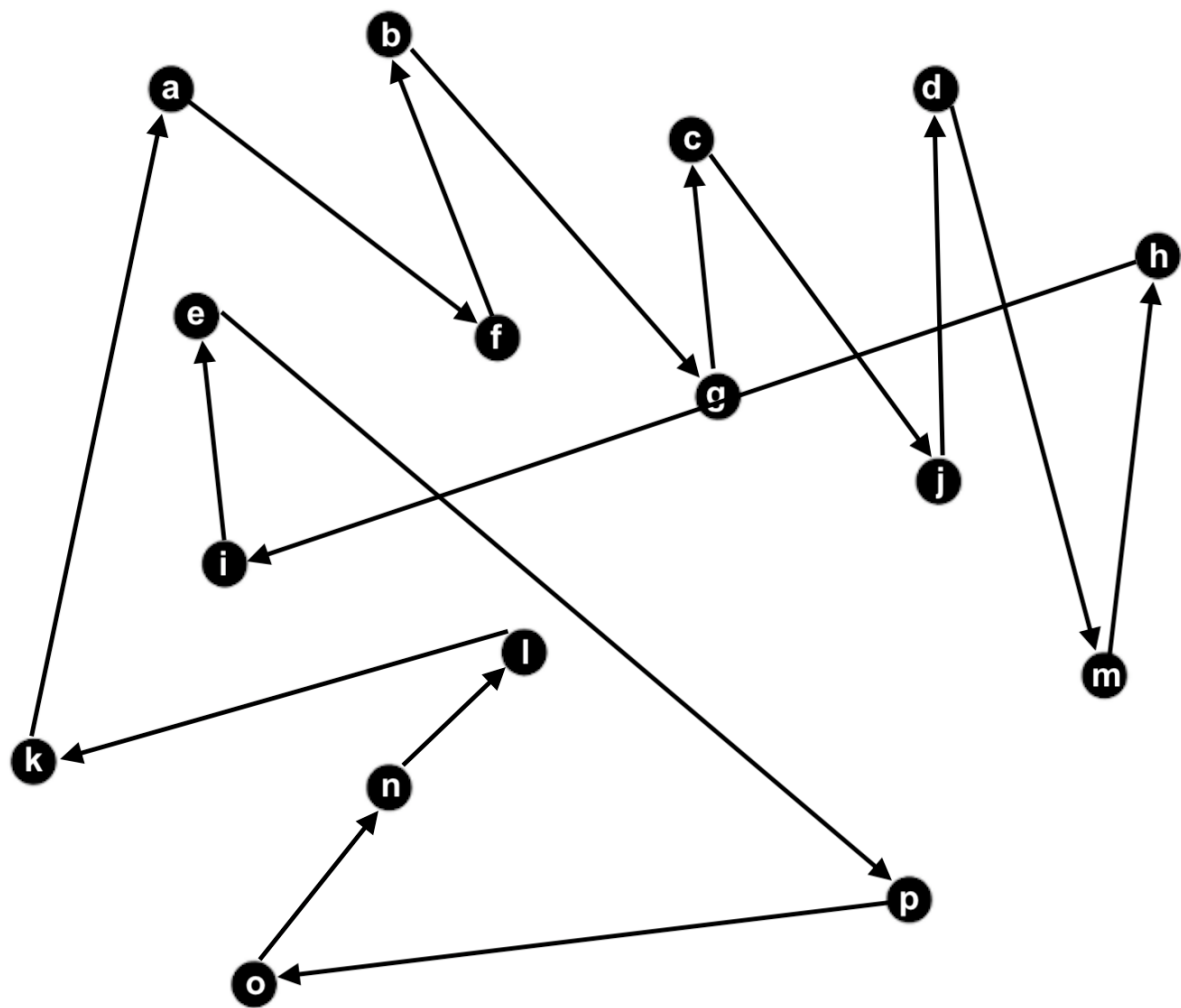
¹ Remember, a connected graph is one in which every vertex can be reached from every other vertex. Intuitively it is in one piece.

3. (10 marks) *The TSP Challenge*

Here is a sample travelling salesman problem. The goal is to find the shortest circuit that visits every node. Since it's a circuit it doesn't matter where you begin as long as you end in the same place.



Submit your answer as a string of letters listing the vertices in the order you would visit them. For example one (nonoptimal) solution would be afbgcdmehieponlka (shown below). Each of the sixteen letters should appear exactly once in your solution, except for the first letter which should reappear at the end (to show that you have returned to your starting point). It will be fun to see how good a solution we can find.



4. (10 marks) Pitcher problems and game trees

- a) Complete the game tree for the pitcher problem begun in the pencast “Game Trees and Pitcher Problems” at <https://www.timtopper.com/Math120.W21/14.61 GT Game Trees.html> If you don’t need to rewatch the pencast you can consult the PDF of the final document at <https://www.timtopper.com/Math120.W21/14.61 GT Game Trees.pdf>
- b) In the pencast I observe that the amounts of water that can be measured range from 0 liters when both pitchers are empty, to 10 liters when both are full. The question remains whether all values within this range, i.e. 1, 2, 3, 4, 5, 6, 7, 8, and 9, can be measured. What conclusion can you draw from your tree?
- c) We already answered this question using a linear modelling approach in Assignment 11. What are the strengths and weaknesses of each approach?

5. (5 marks) *The flow of money: Chilkoot edition*

During the Gold Rush three wealthy tenderfooted travellers arrive at Happy Camp on the Chilkoot trail. Battered, bedraggled, bruised, but still wealthy they pay \$30 for room and board and baths at Kate's "roadhouse" (actually a row of leaky tents), and proceed to take their belongings to their tent. Within a few minutes, Kate decides that even by Gold Rush standards she has overcharged the men and wants to refund some money. She takes \$5 out of her tin box and gives it to her son who is instructed to return it to the three men. On the way, her son, who has never heard of fractions, cannot figure out how to split the \$5 evenly between the three men. To settle the problem, he takes two of the dollars and puts them in his own pocket. When he gets to the tent he returns one dollar to each of the three tenderfeet. Now, if the men originally paid \$10 each for a room, and each received one dollar back, that would mean that they paid \$9 each for the night. Now \$9 times three is \$27, and the son has the \$2 he kept, so that accounts for \$29. Where is the other dollar?

Use a suitable graph to formulate this problem and locate the missing dollar, i.e. use your graph to provide as clear, precise and yet brief a resolution of this puzzle as possible.