

## Example: House Finch Pandemic

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An outbreak of *Mycoplasma Gallisepticum* is infecting a population of approximately 1,100 house finches.

- 2% of susceptible birds become infected each month
- 20% of infected birds die each month, but 9% recover
- 6% die of other causes each month

**What will the time course of this infection be over the next six months? That is, what will the status of the population be month by month?**

From the description we appear to have birds in three states, **Susceptible**, **Infected** and **Dead**.



We are also given some probabilities for transitions between these states:

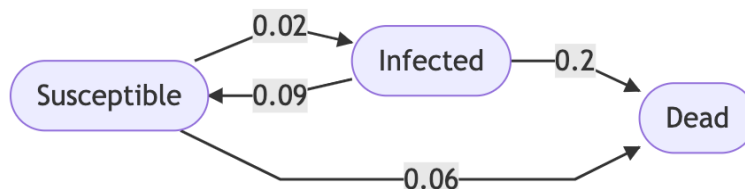
$$p(\text{Susceptible} \rightarrow \text{Infected}) = 0.02$$

$$p(\text{Infected} \rightarrow \text{Dead}) = 0.2$$

$$p(\text{Infected} \rightarrow \text{Susceptible}) = 0.09$$

$$p(\text{Susceptible} \rightarrow \text{Dead}) = 0.06$$

Adding these to our diagram gives us,

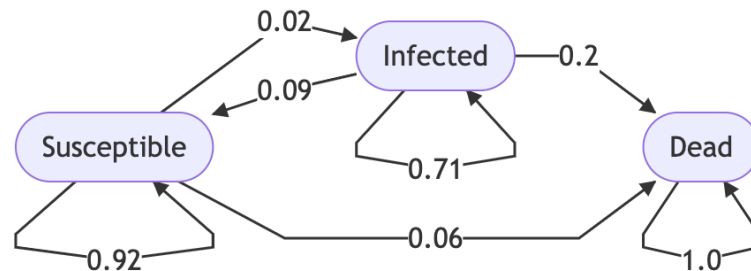


But this is incomplete. One of the fundamental properties of probability is that the probabilities of mutually exclusive events sum to 1.0. In this case it means the weights of all the edges **leaving** a vertex must add to 1.

This means we have to add the following edges:

$$\begin{aligned}
 p(\text{Susceptible} \rightarrow \text{Susceptible}) &= 1 - 0.02 - 0.06 = 0.92 \\
 p(\text{Infected} \rightarrow \text{Infected}) &= 1 - 0.09 - 0.2 = 0.71 \\
 p(\text{Dead} \rightarrow \text{Susceptible}) &= 0 \\
 p(\text{Dead} \rightarrow \text{Infected}) &= 0 \\
 p(\text{Dead} \rightarrow \text{Dead}) &= 1
 \end{aligned}$$

The first two we calculate by subtraction because we know the sum has to be 1. The last three simply follow from what it means to be dead. (We don't normally show edges with weights of 0 because they can't be followed and just clutter the diagram.)



The simplest way to model the time course of the infection would be to use Excel, and we need a transition matrix to do that:

$$\begin{array}{c}
 \begin{array}{c} \text{Susceptible} \\ \text{Infected} \\ \text{Dead} \end{array}
 \begin{array}{c}
 \begin{array}{ccc}
 \text{S} & \text{I} & \text{D} \\
 \left[ \begin{array}{ccc}
 0.92 & 0.02 & 0.06 \\
 0.09 & 0.71 & 0.2 \\
 0 & 0 & 1
 \end{array} \right]
 \end{array}
 \end{array}$$

Then using Excel we can follow the course of the pandemic:

|       |             | S    | I             | D    |    | S              | I    | D    |                | S    | I    | D              |    | S    | I              | D    |     | S              | I    | D    |
|-------|-------------|------|---------------|------|----|----------------|------|------|----------------|------|------|----------------|----|------|----------------|------|-----|----------------|------|------|
|       | Susceptible | 0.92 | 0.02          | 0.06 |    | 0.92           | 0.02 | 0.06 |                | 0.92 | 0.02 | 0.06           |    | 0.92 | 0.02           | 0.06 |     | 0.92           | 0.02 | 0.06 |
|       | Infected    | 0.09 | 0.71          | 0.20 |    | 0.09           | 0.71 | 0.20 |                | 0.09 | 0.71 | 0.20           |    | 0.09 | 0.71           | 0.20 |     | 0.09           | 0.71 | 0.20 |
|       | Dead        | 0    | 0             | 1    |    | 0              | 0    | 1    |                | 0    | 0    | 1              |    | 0    | 0              | 1    |     | 0              | 0    | 1    |
| S     | I           | D    | S             | I    | D  | S              | I    | D    | S              | I    | D    | S              | I  | D    | S              | I    | D   | S              | I    | D    |
| 1100  | 0           | 0    | 1012          | 22   | 66 | 933            | 36   | 131  | 862            | 44   | 194  | 797            | 49 | 255  | 737            | 50   | 312 | 683            | 51   | 367  |
| Start |             |      | After 1 month |      |    | After 2 months |      |      | After 3 months |      |      | After 4 months |    |      | After 5 months |      |     | After 6 months |      |      |