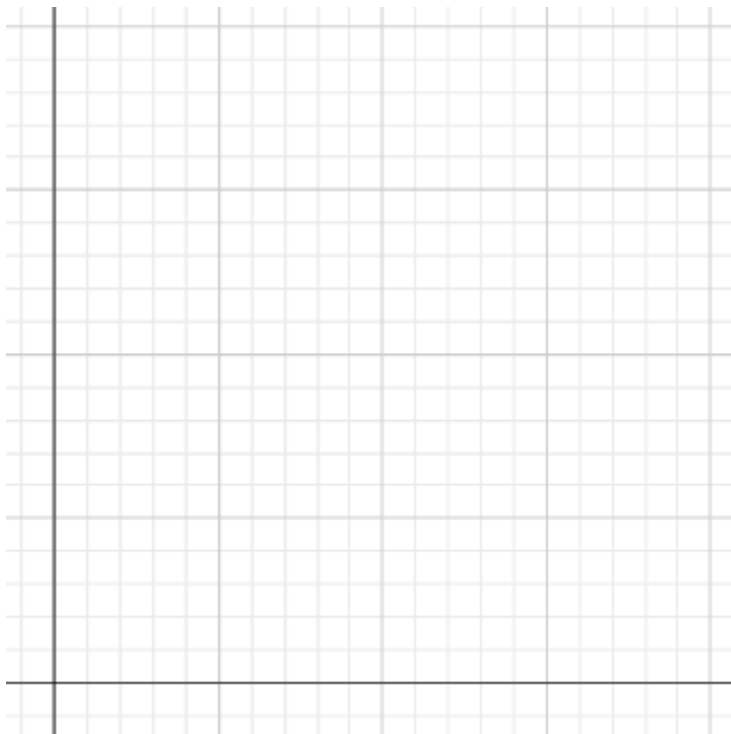


Math 120/ALES 291
Final Examination
April 24, 2020

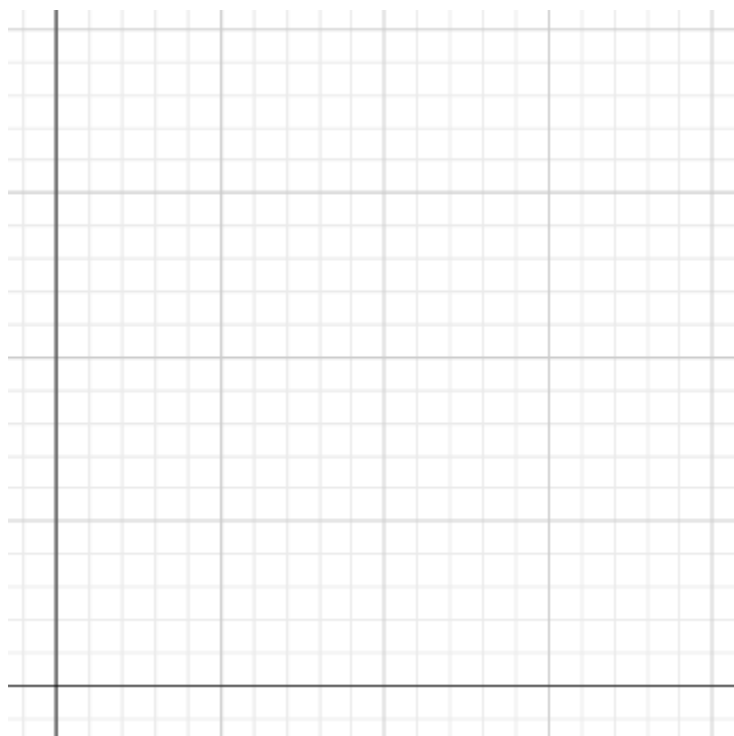
Instructions:

- ◆ You have three (3) hours to complete the examination.
- ◆ You may consult your course notes.
- ◆ You may not seek or accept assistance from anyone else in completing the exam.
- ◆ You may use a scientific calculator, but **not** a graphing calculator, software or online services.
- ◆ The value of all questions is shown — choose well, and use your time wisely!
- ◆ A box **must** be placed around your final answer to each question.
- ◆ Part marks can only be given if your rough work is shown. **No marks** will be given for unsupported answers.
- ◆ If you are unsure of anything, **ask** by email or in the Zoom meeting room (details sent by email).

1. (10 marks) A species of songbird expends 4 calories for each hour spent singing, s , and 10 calories for each hour spent defending territory, d .
- a) If this species expends 60 calories per day on these activities find an equation relating s and d .
- b) Sketch a graph of d vs s .
- c) What do the s and d intercepts tell you about the bird species?



2. (20 marks) A company owns two mines. Mine A produces 1 ton of high-grade ore, 3 tons of medium-grade ore, and 5 tons of low-grade ore each day. Mine B produces 2 tons of each of the three grades of ore each day. The company has an order for 80 tons of high-grade ore, 160 tons of medium-grade ore, and 200 tons of low-grade ore. Suppose it costs \$20,000/day to operate mine A and \$30,000/day to operate mine B. Find the number of days each mine should be operated to supply the order while minimizing costs.



3. (10 marks) For the Simplex tableau below, determine the next pivot element and pivot the tableau about it.

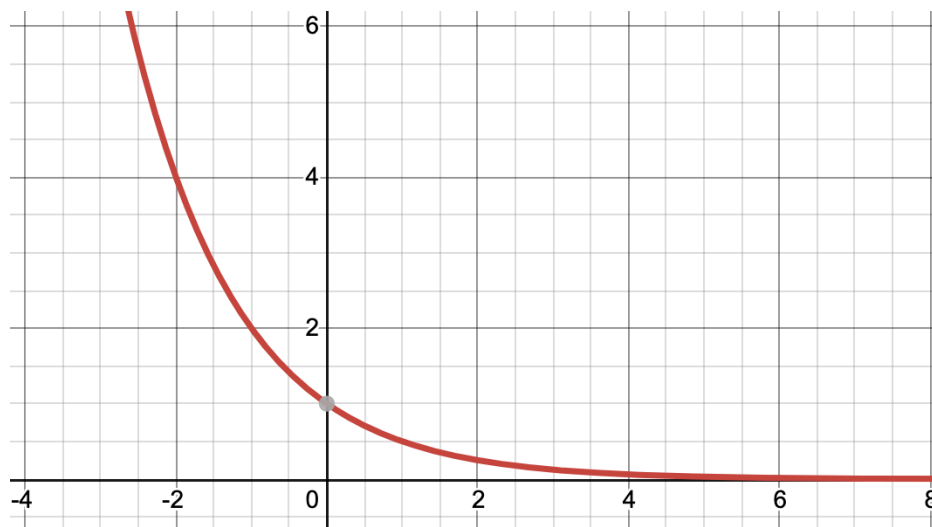
$$\begin{array}{c}
 \begin{array}{cccccc}
 & x & y & u & v & w & p \\
 u & \left[\begin{array}{cccccc}
 2 & 1 & 1 & 0 & 0 & 0 & 7
 \end{array} \right. \\
 v & \left[\begin{array}{cccccc}
 4 & -1 & 0 & 1 & 0 & 0 & 4
 \end{array} \right. \\
 w & \left[\begin{array}{cccccc}
 4 & 3 & 0 & 0 & 1 & 0 & 5
 \end{array} \right. \\
 p & \left[\begin{array}{cccccc}
 -8 & -5 & 0 & 0 & 0 & 1 & 0
 \end{array} \right.
 \end{array} \\
 \begin{array}{cccccc}
 & x & y & u & v & w & p \\
 & \left[\begin{array}{cccccc}
 & & & & & &
 \end{array} \right.
 \end{array}
 \end{array}$$

4. (5 marks) Consider the Simplex Tableau below for a profit maximization problem. There are two products a and b and three resources represented by the slack variables s_1 , s_2 and s_3 :

$$\begin{array}{c}
 \begin{array}{cccccc}
 & a & b & s_1 & s_2 & s_3 & p \\
 b & \left[\begin{array}{cccccc}
 0 & 1 & 10 & 2 & 2 & 0 & 150
 \end{array} \right. \\
 s_1 & \left[\begin{array}{cccccc}
 0 & 0 & 1 & -1 & -2 & 0 & 60
 \end{array} \right. \\
 a & \left[\begin{array}{cccccc}
 1 & 0 & 0 & 1 & 4 & 0 & 120
 \end{array} \right. \\
 p & \left[\begin{array}{cccccc}
 0 & 0 & 0 & 2 & 10 & 1 & 660
 \end{array} \right.
 \end{array}
 \end{array}$$

- Does it represent an optimal solution?
- How many units of each product are being made?
- What is the total profit being earned?
- Which resource is currently preventing profit from increasing? How much would another unit of that resource be worth to the company?

5. (5 marks) What is the equation of the function below?



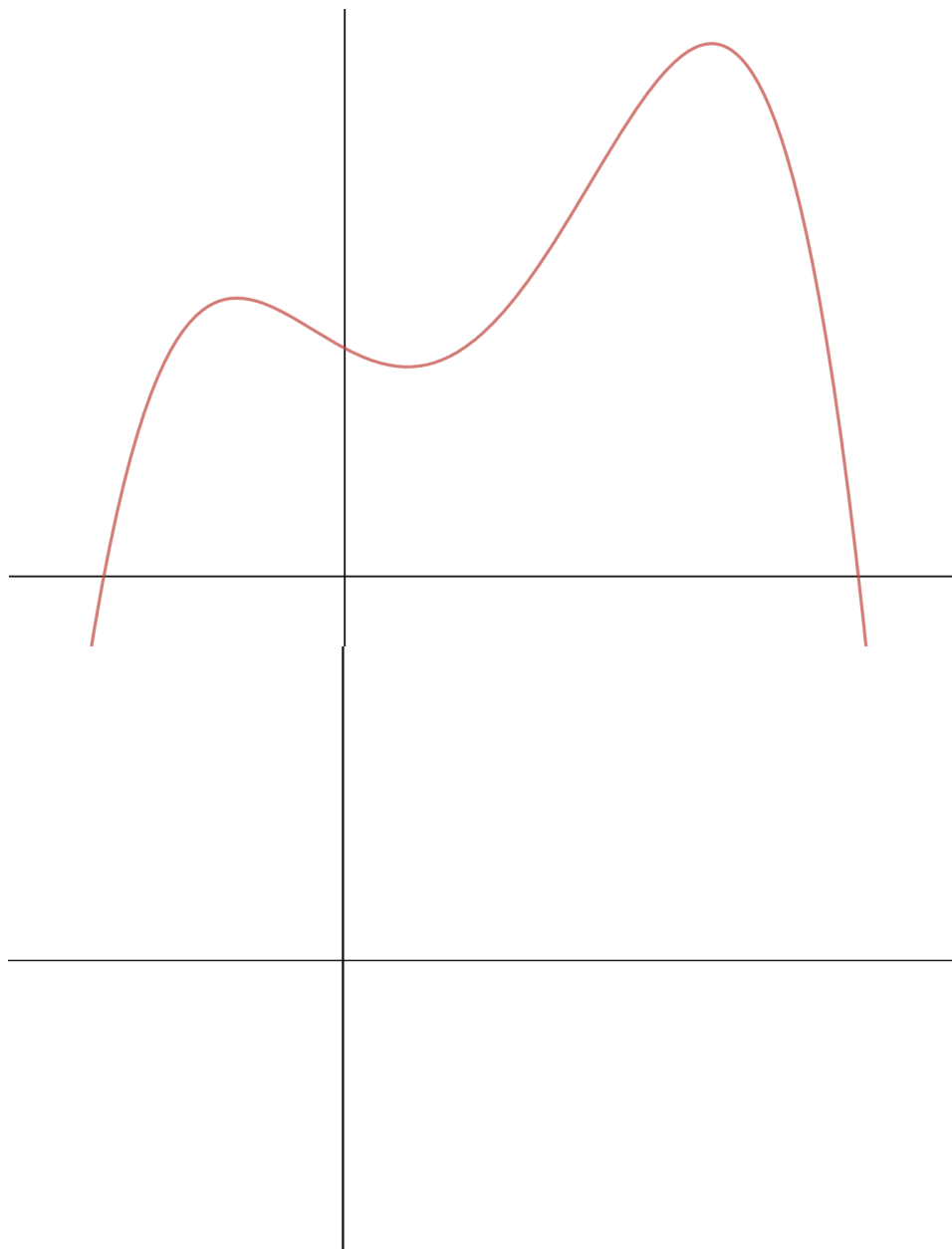
6. (10 marks) In the early going the number of COVID-19 cases in New York state was growing exponentially. At one point there were 7,800 total cases of COVID-19 in New York, when four days earlier there had been 780.

a) How many would there be the next day?

b) How long did they have before there would be 50,000 cases?

c) When was the first case?

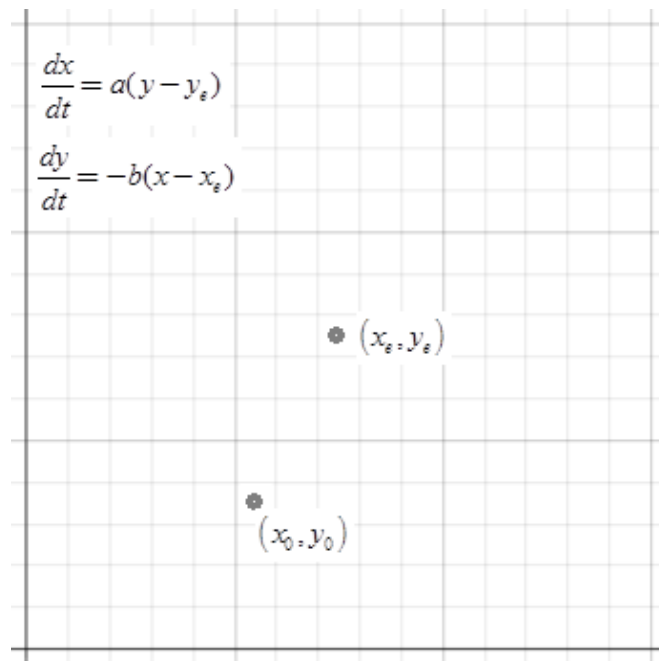
7. (10 marks) Sketch the derivative of this function on the axes below it.



8. (10 marks) Is $y = e^{-t}$ a solution to the differential equation $y'' + 2y' + y = 0$?

9. (10 marks) A small town newspaper sells for \$1 per copy. The cost in dollars of producing x copies is $C(x) = 70 + 0.001x + 0.00005x^2$.
- What does the 70 in the cost function $C(x)$ represent?
 - What number of copies should be published to maximize profit?
 - Suppose the newspaper is taken over by a community group and run as a non-profit venture whose goal is to maximize readership while not losing money (i.e. breaking even). What number of copies would represent their optimum production level?

10. (5 marks) The graph below shows an equilibrium position for the system, (x_e, y_e) , an initial position, (x_0, y_0) , and the derivatives of the x and y quantities. Sketch the system trajectory that results (show at least three steps of the trajectory).



11. (10 marks) Annual per capita consumption of bottled water for the period 1993–2013 can be approximated by $C(t) = 0.05t^2 + 0.4t + 9$ gallons per year ($3 \leq t \leq 23$).

a) At what rate were sales increasing in 1993? In 2003?

b) How many gallons of bottled water were purchased per capita from 1993 to 2013?

12. (10 marks) Your grandmother's coin purse has one penny, one nickel, one dime, one quarter, one loonie and one twonie in it.

a) If she shakes one coin out of it what is the probability that it will be the loonie?

b) If she shakes one coin out of it what is the probability that it will be worth less than a dollar?

c) If she shakes two coins out of it what is the probability that together they will be worth more than 50 cents?

d) If she shakes one coin out of it and then another, what is the probability that the second coin is worth more than the first?

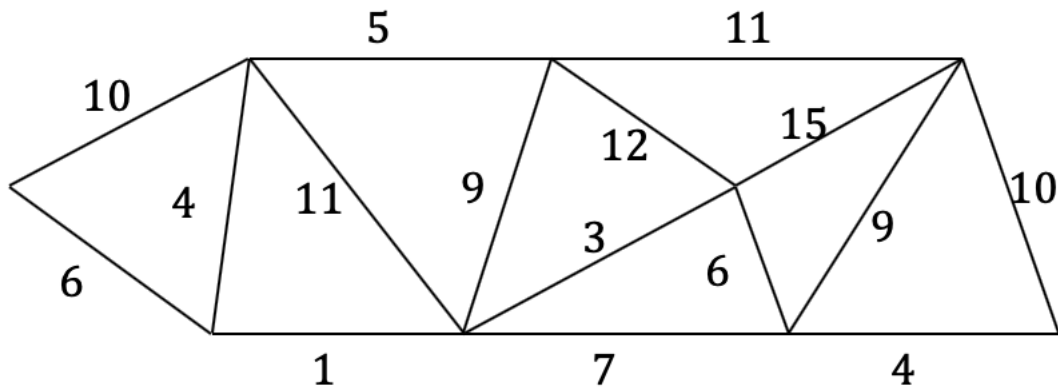
13. (15 marks) Antibody tests for COVID-19 are becoming available. They test if your body has antibodies to COVID-19, i.e. that you have been infected, and are no longer susceptible to getting it. The company Cellex makes an FDA-approved test with a sensitivity of 93.8% and a specificity of 95.6%. Sensitivity is the probability a person tests positive if they have been infected. Specificity is the probability a person tests negative if they have not been infected. In an effort to restart its economy the City of Toronto (where 1.3% of people have been infected) decides to test 100,000 randomly chosen people to find people with antibodies who are thus immune and can safely return to work.

a) How many of the 100,000 people will get a positive test result?

b) How many people with a positive test result will actually be immune?

c) How many people with a positive test result will not actually be immune, and thus at risk of contracting COVID-19 if they return to work?

14. (5 marks) Show the minimal spanning tree for the graph below either by highlighting the edges in the diagram below that are part of it or by drawing just the minimal spanning tree.



15. (10 marks) For the graph whose adjacency matrix is:

	a	b	c	d	e	f	g	h
a	—	9	12	7	4	2	6	3
b	9	—	2	9	6	9	5	7
c	12	2	—	3	11	8	6	5
d	7	9	3	—	9	1	3	7
e	4	6	11	9	—	6	3	4
f	2	9	8	1	6	—	4	9
g	6	5	6	3	3	4	—	8
h	3	7	5	7	4	9	8	—

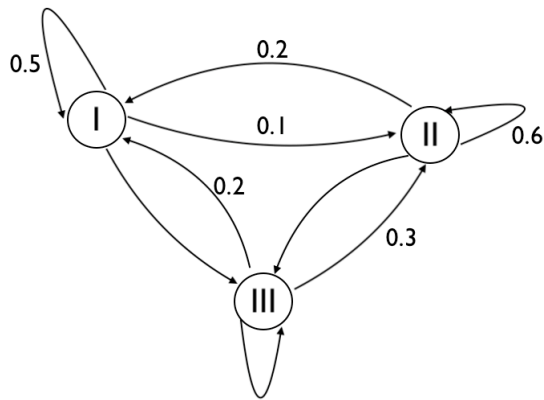
a) What is the shortest route from a to b ?

b) What is the length of this route?

16. (10 marks) For the graph whose adjacency matrix is given below, how many paths of length 3 are there from a to b ?

	a	b	c	d
a	1	1	0	1
b	1	0	1	1
c	1	0	0	0
d	0	1	0	0

17. (5 marks) Consider the directed graph below where the vertices represent states and the weights on the edges represent the probabilities of moving from one state to another.



a) Fill in the missing probabilities.

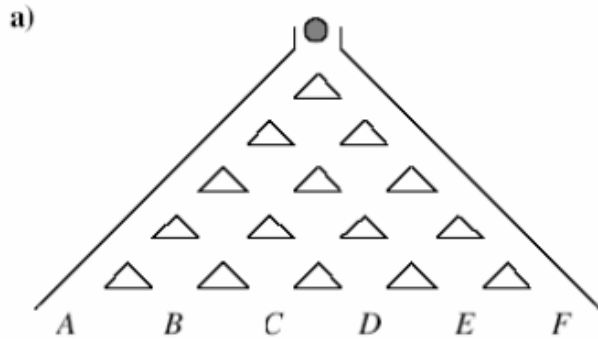
b) Draw the state transition matrix for this system.

18. (10 marks) Calculate the exact equilibrium distribution for the system whose transition matrix is:

	a	b
a	0.7	0.3
b	0.1	0.9

19. (10 marks) Here are diagrams of two pachinko machines in which a marble, , falls from top to bottom bouncing off the triangular bumpers on its way.

- a) How many possible paths are there from the top to each of the exits A , B , C , D , E and F ? (Give me six numbers, one for each of A through F .)



- b) How many possible paths are there the ball could take on its trip from the top to the bottom?

