

Exercise Set 2 Solutions

Manipulating equations:

1. Solve for x :

$$x + 1 = 5 \quad \text{Goal: get } x \text{ alone on one side of } = \text{ sign}$$

$$x + 1 - 1 = 5 - 1 \quad \text{Subtract 1 from both sides to "move" the 1 to the right hand side (RHS)}$$

$$x = 4 \quad \text{Simplify}$$

2. Solve for t :

$$3t - 7 = 14 \quad \text{Goal: Get } t \text{ alone on one side of } = \text{ sign}$$

$$3t - 7 + 7 = 14 + 7 \quad \text{Move the 7 to the RHS by adding 7 to both sides}$$

$$3t = 21 \quad \text{Simplify}$$

$$\frac{3t}{3} = \frac{21}{3} \quad \text{Divide both sides by 3}$$

$$t = 7 \quad \text{Simplify}$$

3. Solve for p :

$$3p + 2 = 6p - 4 \quad \text{Strategy: Gather terms with } p \text{ on one side of } = \text{ and numbers on the other side}$$

$$3p + 2 - 2 = 6p - 4 - 2 \quad \text{Subtract 2 from both sides to "move" 2 to the RHS}$$

$$3p = 6p - 6 \quad \text{Simplify}$$

$$3p - 6p = 6p - 6p - 6 \quad \text{Subtract } 6p \text{ from both sides to move } 6p \text{ to the left hand side (LHS)}$$

$$-3p = -6 \quad \text{Simplify}$$

$$\frac{-3p}{-3} = \frac{-6}{-3} \quad \text{Divide both sides by } -3$$

$$p = 2 \quad \text{Simplify}$$

4. Solve for t :

$$\begin{array}{lll}
 2(t+3) & = & 5(t-1) - 7(t-3) \quad \text{Strategy: Expand, then gather terms as in 3.} \\
 2t+6 & = & 5t-5-7t+21 \quad \text{Expand} \\
 2t+6 & = & -2t+16 \quad \text{Simplify} \\
 2t+6-6 & = & -2t+16-6 \quad \text{Move 6 from LHS to RHS} \\
 2t & = & -2t+10 \quad \text{Simplify} \\
 2t+2t & = & -2t+2t+10 \quad \text{Move -2t from RHS to LHS} \\
 4t & = & 10 \quad \text{Simplify} \\
 \frac{4t}{4} & = & \frac{10}{4} \quad \text{Divide both sides by 4} \\
 t & = & \frac{5}{2} \quad \text{Simplify}
 \end{array}$$

5. Solve for x :

$$\begin{array}{lll}
 \frac{x-3}{2} & = & \frac{2x+4}{5} \quad \text{Strategy: Clear fractions, then gather terms as in 3. and 4.} \\
 \frac{x-3}{2} \cdot 10 & = & \frac{2x+4}{5} \cdot 10 \quad \text{Multiply both sides by 10 (because } 10 = 5 \times 2) \\
 (x-3) \cdot 5 & = & (2x+4) \cdot 2 \quad \text{Cancel denominators (2 and 5) with 10} \\
 5x-15 & = & 4x+8 \quad \text{Expand} \\
 5x-4x & = & 8+15 \quad \text{Gather terms} \\
 x & = & 23 \quad \text{Simplify}
 \end{array}$$

Translating words into symbols

6. One more than twice a certain number.

$$\underbrace{\underbrace{+1}_{\text{One more than}} \underbrace{2 \times x}_{\text{twice a certain number}}}_{2x+1}$$

7. Three less than five times a certain number.

$$\underbrace{\underbrace{-3}_{\text{Three less than}} \underbrace{5 \times x}_{\text{five times a certain number}}}_{5x-3}$$

8. Two numbers whose difference is 10.

$$\underbrace{\underbrace{x \text{ and } y}_{\text{Two numbers}} \underbrace{x-y}_{\text{whose difference is 10}}}_{x-y=10}$$

Solving simultaneous equations

9. Solve:

$$\begin{array}{ll} 1) & 2x - y = 4 \\ 2) & x + 2y = -3 \\ 2') & x = -3 - 2y \quad \text{Solve 2) for } x \text{ by moving } 2y \text{ to the RHS} \\ 1') & 2(-3 - 2y) - y = 4 \quad \text{Substitute 2') into 1)} \\ & -6 - 4y - y = 4 \quad \text{Expand} \\ & -6 - 5y = 4 \quad \text{Simplify} \\ & -5y = 10 \quad \text{Move -6 to RHS} \\ & y = -2 \quad \text{Divide through by -5 to find } y \\ & x = -3 - 2(-2) \quad \text{Substitute } y \text{ into 2')} \\ & x = -3 + 4 = 1 \quad \text{Simplify} \end{array}$$

Therefore $x = 1$ and $y = -2$

10. Solve:

$$\begin{array}{ll} 1) & 2x - 3y = 7 \\ 2) & 3x + y = 5 \\ 2') & y = 5 - 3x \quad \text{Solve 2) for } y \text{ by moving } 3x \text{ to the RHS} \\ 1') & 2x - 3(5 - 3x) = 7 \quad \text{Substitute 2') into 1)} \\ & 2x - 15 + 9x = 7 \quad \text{Expand} \\ & 11x - 15 = 7 \quad \text{Simplify} \\ & 11x = 22 \quad \text{Move -15 to RHS} \\ & x = 2 \quad \text{Divide through by 11 to find } x \\ & y = 5 - 3(2) \quad \text{Substitute } x \text{ into 2')} \\ & y = -1 \quad \text{Simplify} \end{array}$$

Therefore $x = 2$ and $y = -1$

11. Solve:

$$1) \quad 5x + 2y = 3$$

$$2) \quad 2x + 3y = -1$$

$$2x = -1 - 3y$$

Solve 2) for x by moving 3y to the RHS

$$2') \quad x = -\frac{1}{2} - \frac{3}{2}y$$

and then dividing through by 2

$$1') \quad 5\left(-\frac{1}{2} - \frac{3}{2}y\right) + 2y = 3$$

Substitute 2') into 1)

$$-\frac{5}{2} - \frac{15}{2}y + 2y = 3$$

Expand

$$-\frac{5}{2} - \frac{15}{2}y + \frac{4}{2}y = 3$$

Prepare to add fractions

$$-\frac{5}{2} - \frac{11}{2}y = 3$$

Simplify

$$-\frac{11}{2}y = 3 + \frac{5}{2}$$

Move $-\frac{5}{2}$ to RHS

$$-\frac{11}{2}y = \frac{6}{2} + \frac{5}{2}$$

Prepare to add fractions

$$-\frac{11}{2}y = \frac{11}{2}$$

Simplify

$$-y = 1$$

Multiply through by $\frac{2}{11}$

$$y = -1$$

Multiply through by -1

$$x = -\frac{1}{2} - \frac{3}{2}(-1)$$

Substitute y into 2')

$$x = -\frac{1}{2} + \frac{3}{2}$$

Simplify

$$x = \frac{2}{2} = 1$$

Simplify

Therefore $x = 1$ and $y = -1$

Artificial problems

12. The sum of two numbers is 21 and one number is twice the other. What are the numbers?

$$\underbrace{\text{The sum of two numbers}}_{x+y} \text{ is } \underbrace{21}_{=21} \quad \text{and}$$
$$\underbrace{\text{one number}}_x \text{ is } \underbrace{\text{twice the other.}}_{=2y}$$

So we have the two equations:

$$1) \quad x + y = 21$$

$$2) \quad x = 2y$$

$$2y + y = 21 \quad \text{Substitute 2) into 1)}$$

$$3y = 21 \quad \text{Simplify}$$

$$\frac{3y}{3} = \frac{21}{3} \quad \text{Divide through by 3.}$$

$$y = 7 \quad \text{Simplify}$$

$$x = 2y = 2 \cdot 7 = 14 \quad \text{Substitute y value into 2) to find x}$$

Therefore the numbers are 7 and 14.

13. The sum of three consecutive whole numbers is 24. Find the numbers.

$$\underbrace{\text{The sum of three consecutive whole numbers}}_{x+(x+1)+(x+2)} \text{ is } \underbrace{24}_{=24}$$

So we have the single equation:

$$x + (x+1) + (x+2) = 24$$

$$x + x + 1 + x + 2 = 24 \quad \text{Remove brackets}$$

$$3x + 3 = 24 \quad \text{Simplify}$$

$$3x = 21 \quad \text{Move 3 from LHS to RHS and subtract}$$

$$\frac{3x}{3} = \frac{21}{3} \quad \text{Divide through by 3.}$$

$$x = 7 \quad \text{Simplify}$$

Therefore the numbers are 7, 8 and 9.

14. Betty has 50 coins, all in nickels and dimes, worth \$3.50. How many nickels does she have?

$$\underbrace{\text{Betty has 50 coins, all in nickels and dimes,}}_{n+d=50} \underbrace{\text{worth \$3.50,}}_{n,d} \underbrace{5n+10d=350}$$

So we have the two equations:

$$1) \quad n + d = 50$$

$$2) \quad 5n + 10d = 350$$

$$1') \quad d = 50 - n$$

Solve 1) for d by moving n to RHS

$$5n + 10(50 - n) = 350$$

Substitute 1') into 2)

$$5n + 500 - 10n = 350$$

Expand

$$-5n + 500 = 350$$

Simplify

$$-5n = -150$$

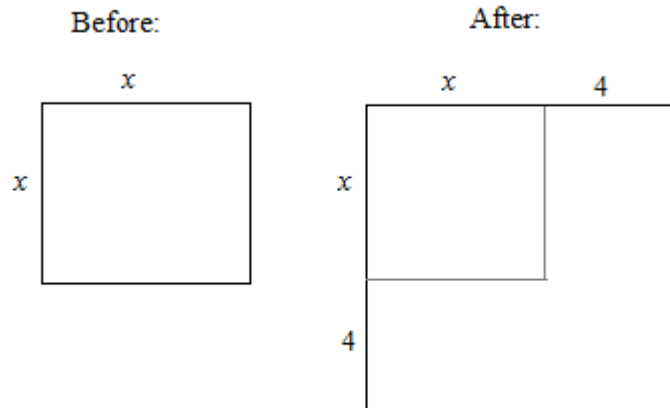
Move 500 to RHS and subtract

$$n = 30$$

Divide through by -5

15. John made each side of his garden plot 4 feet longer. This increased its area by 64 square feet. What were its original dimensions?

Let's draw a picture to try and visualize the situation:



From the diagrams we can see that:

$$\text{Area before} = x \cdot x$$

$$\text{Area after} = (x + 4) \cdot (x + 4)$$

And we are told that: Area after = Area before + 64, so:

$$(x + 4) \cdot (x + 4) = x \cdot x + 64$$

$$x^2 + 4x + 4x + 16 = x^2 + 64 \quad \text{Expand}$$

$$x^2 + 8x + 16 = x^2 + 64 \quad \text{Simplify}$$

$$8x + 16 = 64 \quad \text{Subtract } x^2 \text{ from both sides}$$

$$8x = 48 \quad \text{Move 16 to RHS}$$

$$x = 6 \quad \text{Divide through by 8}$$

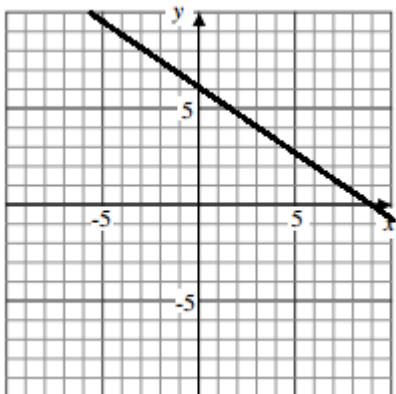
So the original dimensions were 6 feet by 6 feet.

Graphing lines

16. Graph the line $2x + 3y = 18$

Set x to 0 to find the y intercept: $2(0) + 3y = 18 \Rightarrow 3y = 18 \Rightarrow y = 6$

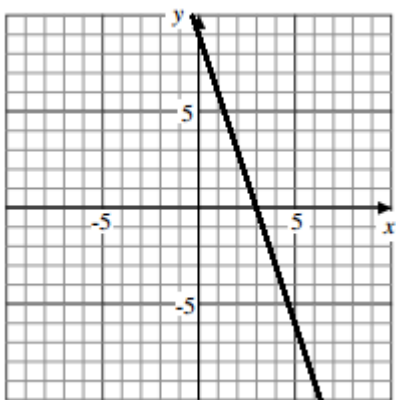
Set y to 0 to find the x intercept: $2x + 3(0) = 18 \Rightarrow 2x = 18 \Rightarrow x = 9$



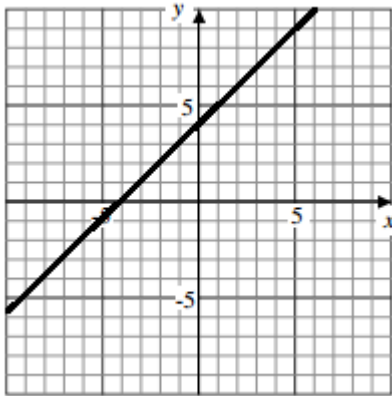
17. Graph the line $12x + 4y = 36$

Set x to 0 to find the y intercept: $12(0) + 4y = 36 \Rightarrow 4y = 36 \Rightarrow y = 9$

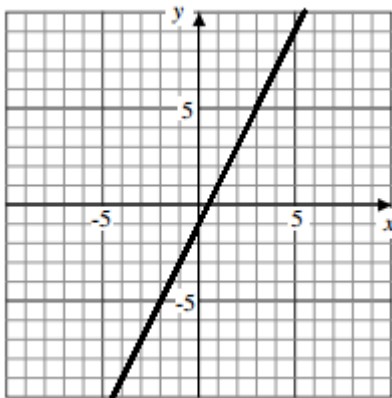
Set y to 0 to find the x intercept: $12x + 4(0) = 36 \Rightarrow 12x = 36 \Rightarrow x = 3$



18. Graph the line $y = x + 4$. In this form we can see that the line's y intercept is 4 and that it has slope 1 so we can sketch it without further calculations.



19. Graph the line $y = 2x - 1$. In this form we can see that the line's y intercept is -1 and that it has slope 2 so we can sketch it without further calculations.



20. Graph the line $y = -2x + 6$. In this form we can see that the line's y intercept is +6 and that it has slope -2 so we can sketch it without further calculations.

