

Homework set 4.1: Linear Programming Skills

Note: You will find the solution to the problem on each page on the next page, so you can check your work at each step as you proceed.

Problem 1. Dairy farm

Organize the information

A dairy farmer concludes that his small herd of cows will need at least 4550 pounds of protein in their Winter feed, at least 26,880 pounds of digestible nutrients (TDN), and at least 43,200 international units (IUs) of vitamin A. Each pound of alfalfa hay provides 0.13 pounds of protein, 0.48 pounds of TDN, and 2.16 IUs of vitamin A. Each pound of ground ears of corn supplies 0.065 pounds of protein, 0.96 pounds of TDN and no vitamin A. Alfalfa hay costs \$1 per 100-pound sack. Ground ears of corn costs \$1.60 per 100-pound sack. The farmer would like to meet the herd's nutritional needs at the least cost.

Fill in the following chart:

	Alfalfa	Corn	Requirement
Protein			
TDN			
Vitamin A			
Cost/lb			(Leave blank)

Formulate the problem (i.e. find the objective function and constraints)

	Alfalfa	Corn	Requirement
Protein	0.13 pounds	0.065 pounds	4,550 pounds
TDN	0.48 pounds	0.96 pounds	26,880 pounds
Vitamin A	2.16 IUs	0	43.200 IUs
Cost/lb	\$0.01	\$0.016	

Objective function:

Use the information in the bottom row of the table to express the cost of buying a pounds of alfalfa hay and c pounds of corn.

Constraints:

Each of the first three rows in the table represent a constraint. Express each one as an inequality using a and c .

Protein:

TDN:

Vitamin A:

What formal constraints are there on the values of a and c ?

Graph the inequalities

Dairy farmer cont'd:

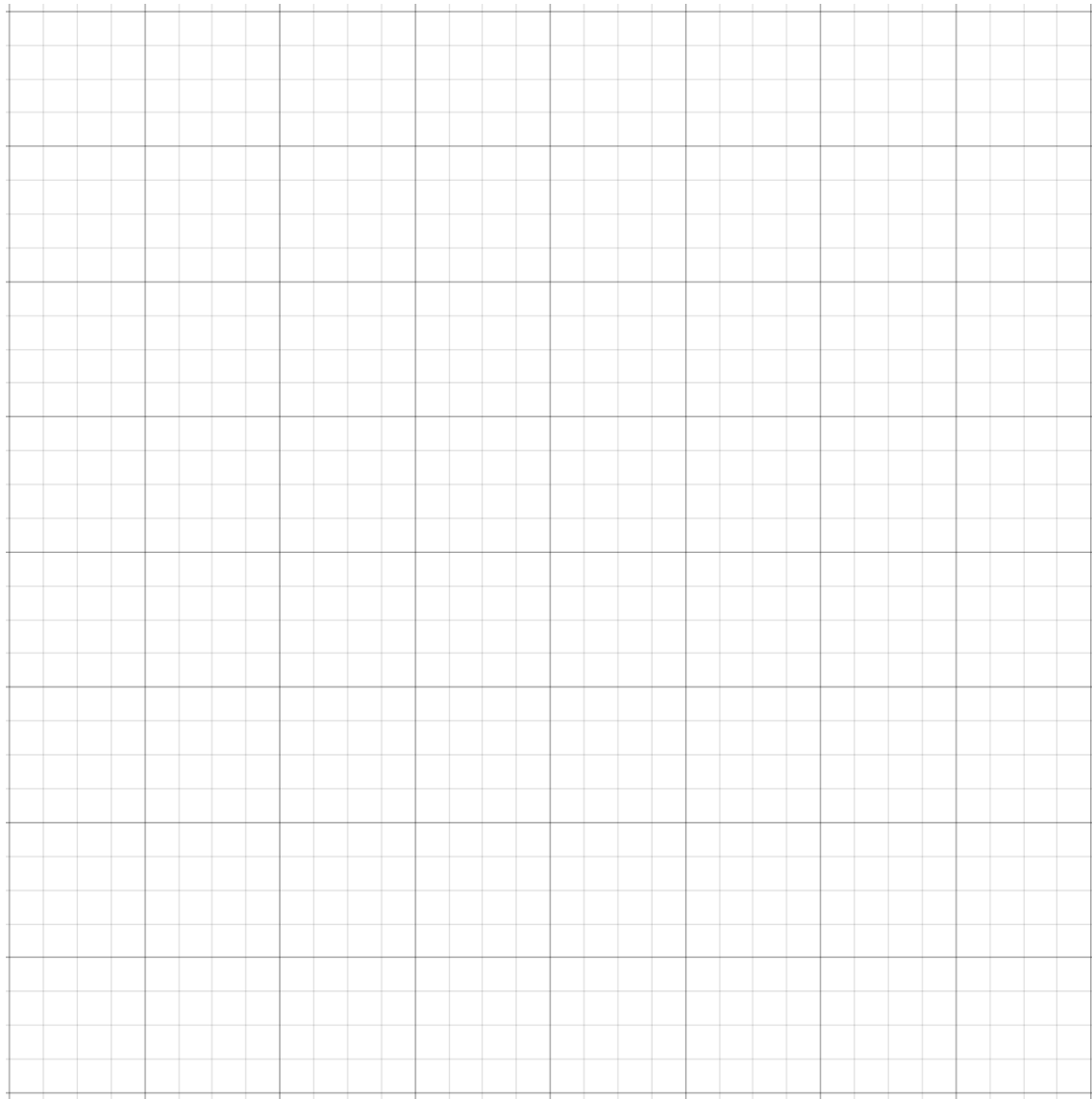
Graph the inequalities below. (Remember: graph the line and then shade the region NOT in the feasible set.)

Protein: $0.13a + 0.065c \geq 4,550$

TDN: $0.48a + 0.96c \geq 26,880$

Vitamin A: $2.16a \geq 43,200$

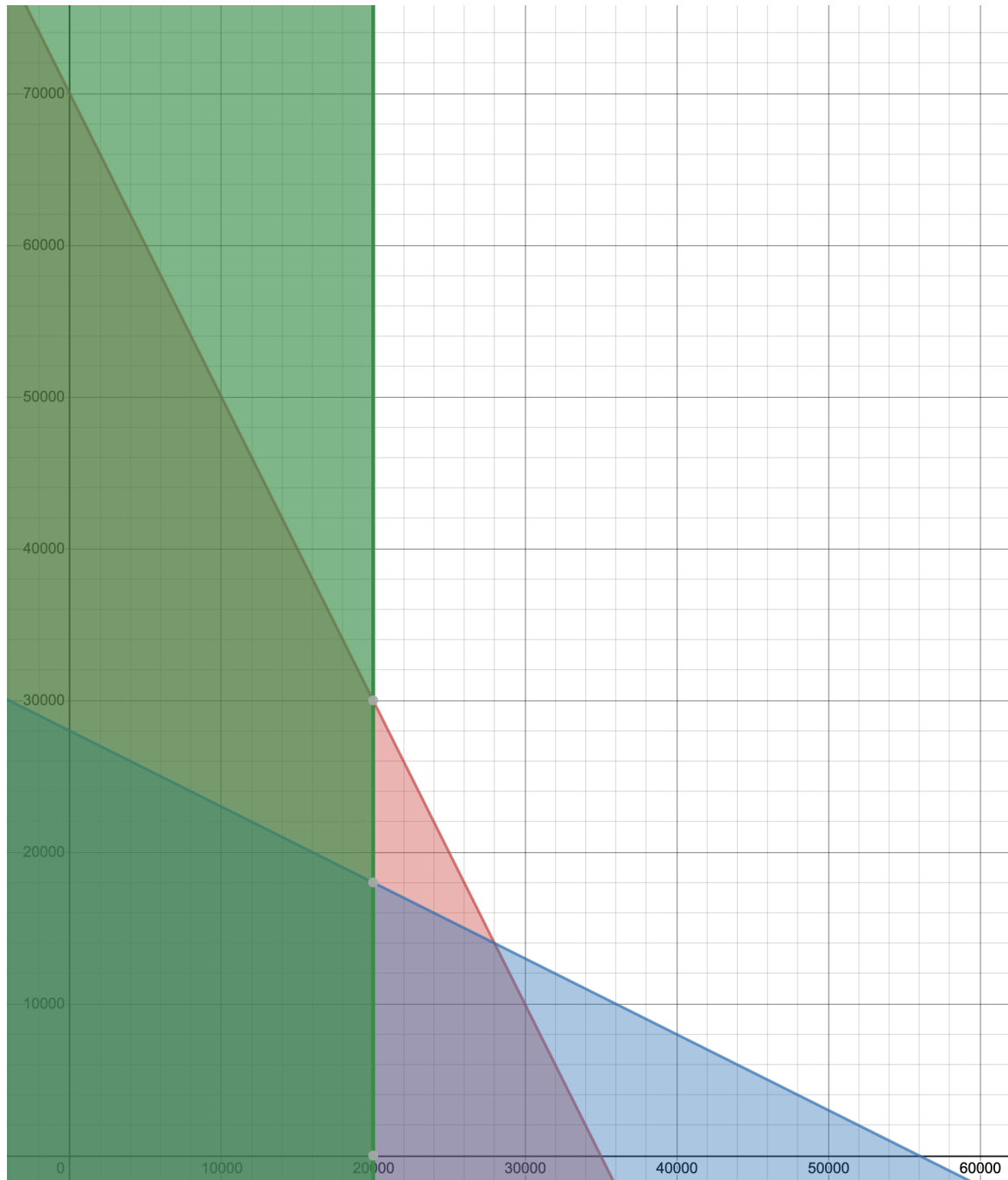
Formal: $c \geq 0$



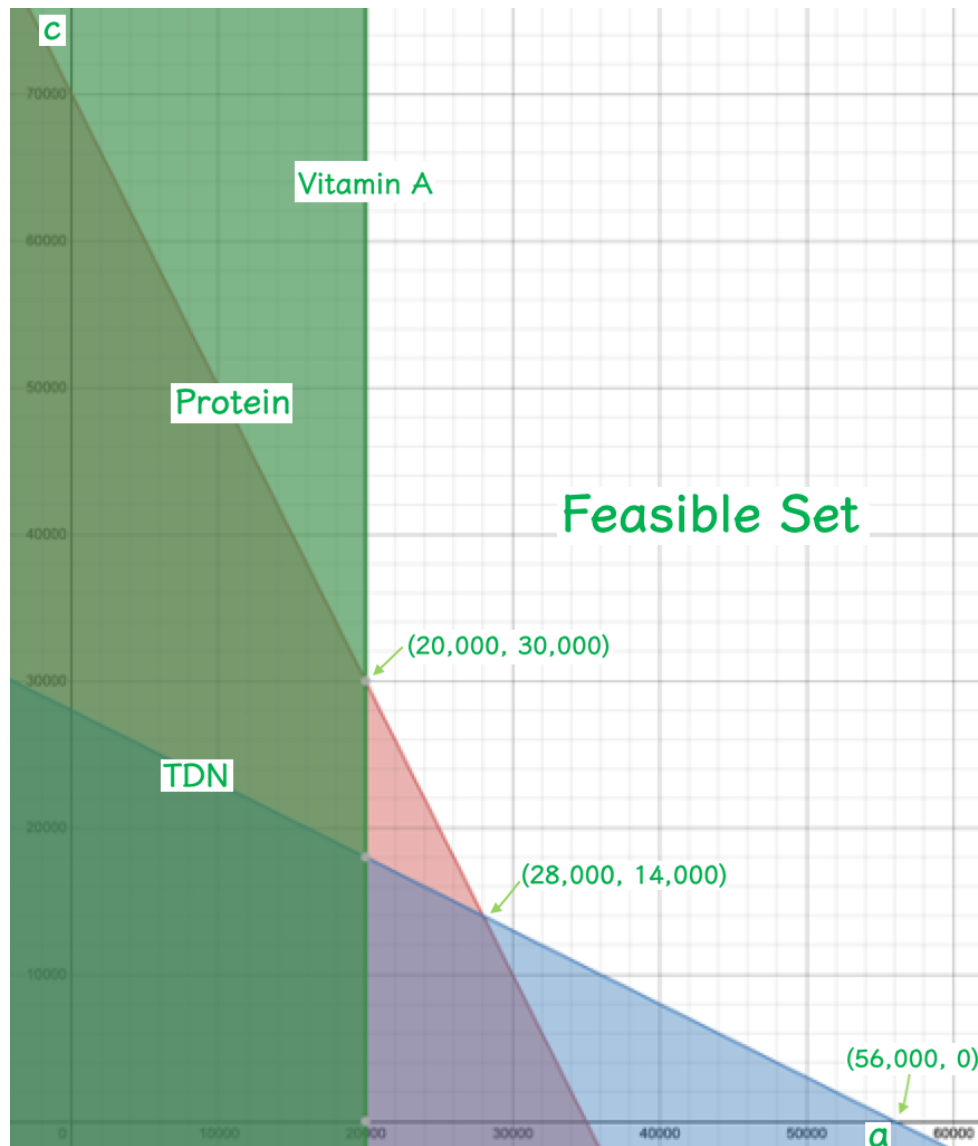
Identify the feasible set and find the necessary vertex coordinates

Label each of the lines below by the constraint it represents, i.e. Protein, TDN and Vitamin A.

Which lines' intersections do we need to find? Find their coordinates.



Calculate the value of the objective function and identify the optimal solution



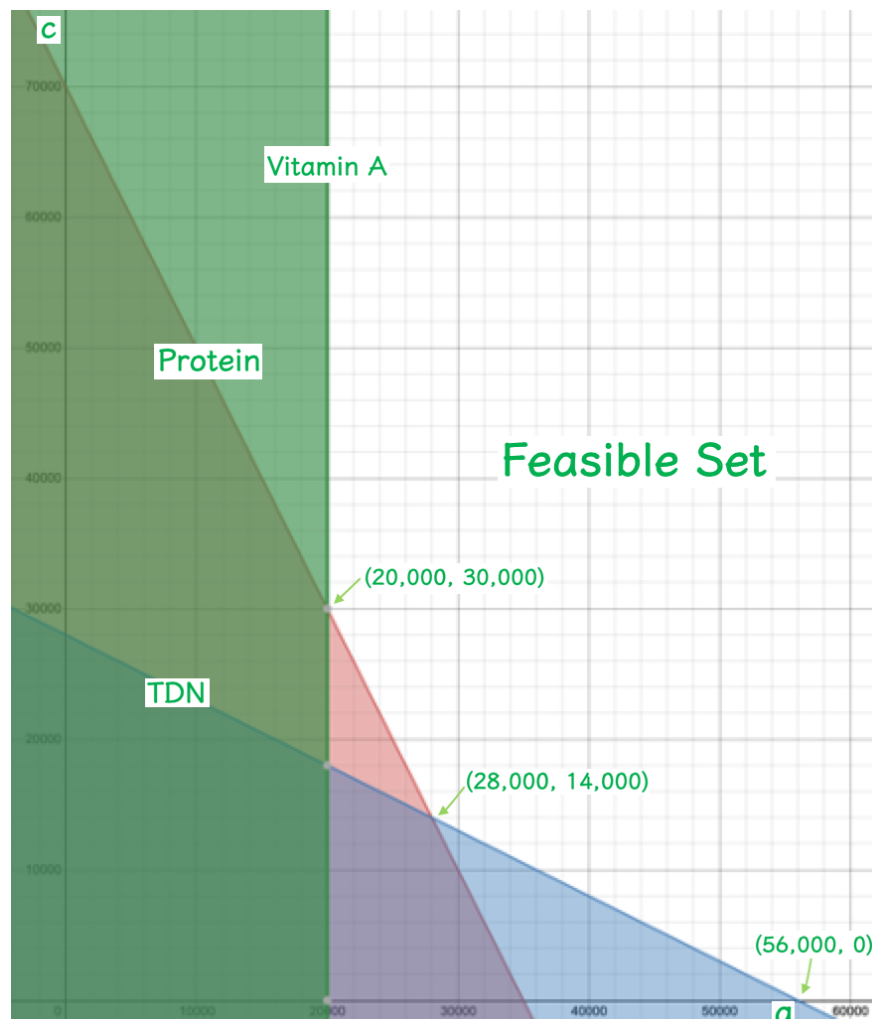
Calculate the value of the objective function, $\text{cost} = 0.01a + 0.016c$, at each of the vertices:

Vertex	Cost
$(20,000, 30,000)$	
$(28,000, 14,000)$	
$(56,000, 0)$	

Which vertex represents the optimal solution?

What is your final advice to the farmer?

Final answer



Calculate the value of the objective function $\text{cost} = 0.01a + 0.016c$ at each of the vertices:

Vertex	Cost
(20,000, 30,000)	$0.01(20,000) + 0.016(30,000) = 680$
(28,000, 14,000)	$0.01(28,000) + 0.016(14,000) = 504$
(56,000, 0)	$0.01(56,000) + 0.016(0) = 560$

Which vertex represents the optimal solution? (28,000, 14,000)

What is your final advice to the farmer? To minimize his cost while still meeting his herd's nutritional needs he should order 28,000 lbs of alfalfa and 14,000 lbs of corn. The total cost will be \$504.

Problem 2. Clothing manufacturer

Organize the information

A *clothing manufacturer* makes denim and hooded fleece jackets. Each denim jacket requires 2 labour-hours for cutting the pieces, 2 labour-hours for sewing and 1 labour-hour for finishing. Each hooded fleece jacket requires 1 labour-hour for cutting, 4 labour-hours for sewing and 1 labour-hour for finishing. There are 42 labour-hours for cutting, 90 labour-hours for sewing and 27 labour-hours for finishing available each day. The profit is \$9 per denim jacket and \$5 per hooded fleece jacket. The manufacturer would like to maximize its profit.

Fill in the following chart:

	Denim	Hooded Fleece	Available hours
Cutting			
Sewing			
Finishing			
Profit			(Leave blank)

Formulate the problem (i.e. find the objective function and constraints)

	Denim	Hooded Fleece	Available hours
Cutting	2 hours	1 hour	42 hours
Sewing	2 hours	4 hours	90 hours
Finishing	1 hour	1 hour	27 hours
Profit	\$9	\$5	

Objective function:

Use the information in the bottom row of the table to express the profit made by producing d denim jackets and h hooded fleece jackets.

Constraints:

Each of the first three rows in the table represent a constraint. Express each one as an inequality using d and h .

Cutting:

Sewing:

Finishing:

What formal constraints are there on the values of d and h ?

Graph the inequalities

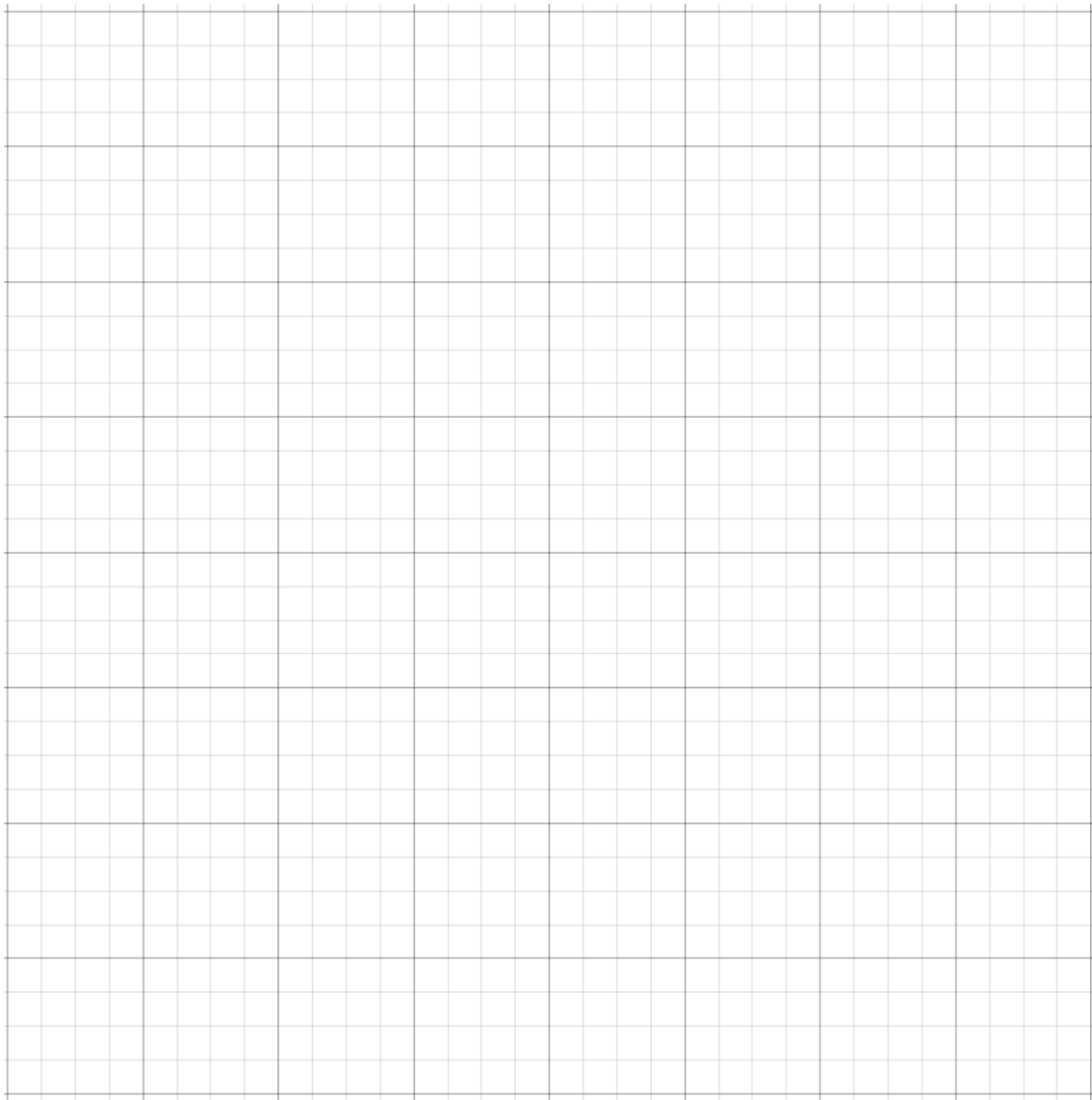
Graph the inequalities below. (Remember: graph the line and then shade the region NOT in the feasible set.)

Cutting: $2d + h \leq 42$

Sewing: $2d + 4h \leq 90$

Finishing: $d + h \leq 27$

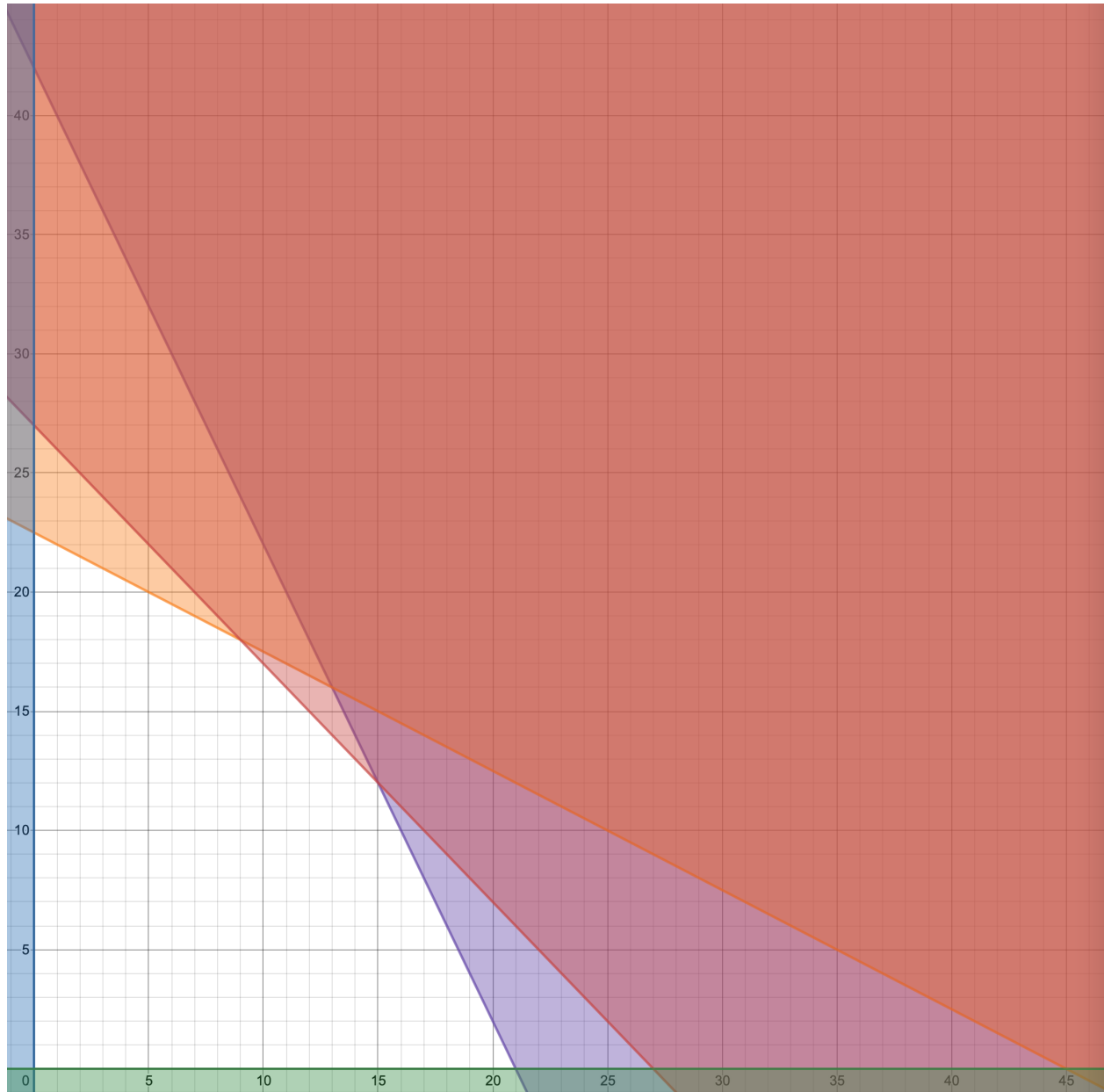
Formal: $d \geq 0, h \geq 0$



Identify the feasible set and find the necessary vertex coordinates

Label each of the lines below by the constraint it represents, i.e. Cutting, Sewing or Finishing.

Which lines' intersections do we need to find? Find their coordinates.



Calculate the value of the objective function and identify the optimal solution



Calculate the value of the objective function profit = $9d + 5h$ at each of the vertices:

Vertex	Profit
$(0, 0)$	
$(0, 22.5)$	
$(9, 18)$	
$(15, 12)$	
$(21, 0)$	

Which vertex represents the optimal solution?

What is your final advice to the manufacturer?

Final answer



Calculate the value of the objective function $\text{profit} = 9d + 5h$ at each of the vertices:

Vertex	Profit
$(0, 0)$	$9(0) + 5(0) = 0$
$(0, 22.5)$	$9(0) + 5(22.5) = 112.5$
$(9, 18)$	$9(9) + 5(18) = 171$
$(15, 12)$	$9(15) + 5(12) = 195$
$(21, 0)$	$9(21) + 5(0) = 189$

Which vertex represents the optimal solution? $(15, 12)$

What is your final advice to the manufacturer? To maximize their profit they should manufacture 15 denim jackets and 12 hooded fleeces. The expected profit is \$195.