

Exercise set 5.1: Simplex Method Skills for Problems in Standard Form

Write the initial simplex tableau for each of these linear programming problems.

1. Maximize $p = 3a + 4b$ subject to the constraints $a + b \leq 20$, $a + 2b \leq 25$, $a \geq 0, b \geq 0$

$\textcircled{1} \quad p - 3a - 4b = 0$
 $\textcircled{2} \quad a + b + u = 20$
 $\textcircled{3} \quad a + 2b + v = 25$

$\begin{matrix} & a & b & u & v & p \\ u & \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 & 20 \\ 1 & 2 & 0 & 1 & 0 & 25 \\ \hline -3 & -4 & 0 & 0 & 1 & 0 \end{array} \right] \end{matrix}$

2. Maximize $C = 4x + 2y + z$ subject to the constraints $x - y + 2z \leq 10$, $2x + y + 3z \leq 12$, $x \geq 0, y \geq 0, z \geq 0$

$\textcircled{1} \quad C - 4x - 2y - z = 0$
 $\textcircled{2} \quad x - y + 2z + u = 10$
 $\textcircled{3} \quad 2x + y + 3z + v = 12$

$\begin{matrix} & x & y & z & u & v & C \\ u & \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 0 & 0 & 10 \\ 2 & 1 & 3 & 0 & 1 & 0 & 12 \\ \hline -4 & -2 & -1 & 0 & 0 & 1 & 0 \end{array} \right] \end{matrix}$

3. Maximize $f = 3a + 5b + 12c$ subject to the constraints $4a + 6b - 7c \leq 16$, $3a + 2b \leq 11$, $9b + 3c \leq 21$, $a \geq 0, b \geq 0, c \geq 0$

$\textcircled{1} \quad f - 3a - 5b - 12c = 0$
 $\textcircled{2} \quad 4a + 6b - 7c + u = 16$
 $\textcircled{3} \quad 3a + 2b + v = 11$
 $\textcircled{4} \quad 9b + 3c + w = 21$

$\begin{matrix} & a & b & c & u & v & w & f \\ u & \left[\begin{array}{cccc|c} 4 & 6 & -7 & 1 & 0 & 0 & 0 & 16 \\ 3 & 2 & 0 & 0 & 1 & 0 & 0 & 11 \\ 0 & 9 & 3 & 0 & 0 & 1 & 0 & 21 \\ \hline -3 & -5 & -12 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \end{matrix}$

Circle the pivot column and pivot element for each of these initial simplex tableaux.

4.
$$\begin{array}{c|cccccc} & a & b & u & v & p & \\ \hline u & 1 & 1 & 1 & 0 & 0 & 20 \\ v & 1 & 2 & 0 & 1 & 0 & 25 \\ p & -3 & -4 & 0 & 0 & 1 & 0 \end{array}$$

① ↑ column

② Ratios:
 $20/1 = 20$
 $25/2 = 12.5 \leftarrow \text{row } ③$
 —
 Not considered
 $\therefore$ its the objective function.

5.
$$\begin{array}{c|cccccc} & x & y & z & u & v & C \\ \hline u & 1 & -1 & 2 & 1 & 0 & 0 & 10 \\ v & 2 & 1 & 3 & 0 & 1 & 0 & 12 \\ C & -4 & -2 & -1 & 0 & 0 & 1 & 0 \end{array}$$

↑ column

Ratios:
 $10/1 = 10$
 $12/2 = 6 \leftarrow \text{row}$
 —

6.
$$\begin{array}{c|cccccc} & a & b & c & u & v & w & f \\ \hline u & 4 & 6 & -7 & 1 & 0 & 0 & 16 \\ v & 3 & 2 & 0 & 0 & 1 & 0 & 11 \\ w & 0 & 9 & 3 & 0 & 0 & 1 & 21 \\ f & -3 & -5 & -12 & 0 & 0 & 0 & 1 & 0 \end{array}$$

↑ column

Ratios:
 $16/-7$ negative!
 $11/0$ undefined!
 $21/3 = 7 \leftarrow \text{row}$
 —
 so not considered

7.
$$\begin{array}{c|cccccc} & x & y & z & u & v & w & p \\ \hline u & 4 & 1 & 5 & 1 & 0 & 0 & 20 \\ v & 1 & 2 & 4 & 0 & 1 & 0 & 50 \\ w & 4 & 10 & 1 & 0 & 0 & 1 & 32 \\ p & -16 & -4 & 20 & 0 & 0 & 0 & 1 & 0 \end{array}$$

↑ column

Ratios:
 $20/4 = 5 \leftarrow \text{row}$
 $50/1 = 50$
 $32/4 = 8$
 —

8. Assume the tableau below represent a manufacturing system where the goal is to maximize the profit p , by manufacturing products x, y, z , subject to constraints on the resources U, V and W.

Offer an *interpretation* of each of the tableaux below, i.e. what production plan does each one correspond to?

	x	y	z	u	v	w	p
u	4	1	10	1	0	0	40
v	1	2	4	0	1	0	50
w	4	10	1	0	0	1	32
p	-16	-4	-20	0	0	0	1

$\Rightarrow x, y, z = 0$
 $u = 40$
 $v = 50$
 $w = 32$
 $p = 0$
 $\therefore$ not making anything.
 No profit.
 All resources unused.

	x	y	z	u	v	w	p
x	0.4	0.1	1	0.1	0	0	4
v	-0.6	1.6	0	-0.4	1	0	34
w	3.6	9.9	0	0	0	1	28
p	-8	-2	2	0	0	0	1

$x = 4$
 $v = 34$
 $w = 28$
 $p = 80$
 Making 4 units of x for profit of 80.
 All of resource U used but have 34 V and 28 W available

	x	y	z	u	v	w	p
x	1	-1	1	0.11	0	-0.11	0
v	0	3.25	0	-0.42	1	0.17	0
z	1	2.75	0	-0.02	0	0.28	0
p	0	20	0	1.8	0	2.22	1

Optimum reached!

Making 0.89 units of x , and 7.8 units of z , for profit of 142.2.

38.7 units of resource V going unused.

What is the interpretation of the number 1.8 and 2.22 in the bottom row of the final tableau?

Shadow prices. Another unit of U would be worth 1.8.

" " " W " " " 2.22.

If two more units of the resource W were available how would the production plan and profit change? Use sensitivities to get new values:

$$x = 0.89 + 2(-0.11) = 0.67$$

$$v = 38.7 + 2(0.17) = 39.04$$

$$z = 7.8 + 2(0.28) = 8.36$$

$$p = 142.22 + 2(2.22) = 146.66$$

What about two more units of V?

$$x = 0.89 + 2(0) = 0.89$$

$$v = 38.7 + 2(1) = 40.7$$

$$z = 7.8 + 2(0) = 7.8$$

In words:
 Decrease production of x by 0.22 units.
 Increase production of

$$P = 142.22 + 2(0) = 142.22$$

Neither production of x or z , nor profit P change. We already have a surplus of v so getting more is no help.

z by 0.56 units

Increase surplus of v by 0.34 units.

Profit increases by 4.44