

Combining functions 1: Products and quotients

$$x \sin x \quad \frac{e^x}{x} \quad \frac{(x^2+1)e^x}{\sin x + 2x}$$

$$\boxed{\frac{d}{dx} [f(x)g(x)] = f(x)g'(x) + g(x)f'(x)} \quad (fg)' = f \cdot g' + g \cdot f'$$

$$\begin{aligned} \text{eg } \frac{d}{dx} [x \sin x] &= x \cdot \frac{d}{dx} [\sin x] + \sin x \cdot \frac{d}{dx} [x] \\ &= x \cdot \cos x + \sin x \cdot 1 \\ &= x \cos x + \sin x \end{aligned}$$

$$\boxed{\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}} \quad \left(\frac{f}{g} \right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$$

$$\begin{aligned} \text{eg } \frac{d}{dx} \left[\frac{e^x}{x} \right] &= \frac{x \cdot \frac{d}{dx} [e^x] - e^x \cdot \frac{d}{dx} [x]}{(x)^2} \\ &= \frac{x \cdot e^x - e^x \cdot 1}{x^2} \\ &= \frac{x e^x - e^x}{x^2} \end{aligned}$$

$$\text{eg } y = \frac{(x^2+1)e^x}{\sin x + 2x} \quad (x^2+1) \cdot e^x + e^x(2x) \quad \cos x + 2$$

$$y' = \frac{(\sin x + 2x) \frac{d}{dx} [(x^2+1)e^x] - (x^2+1)e^x \frac{d}{dx} [\sin x + 2x]}{(\sin x + 2x)^2}$$

$$= \frac{(\sin x + 2x) \left((x^2+1)e^x + e^x(2x) \right) - (x^2+1)e^x (\cos x + 2)}{(\sin x + 2x)^2}$$