

Territorial Birds



Consider a territorial bird that harvests food from a territory that it defends. The amount of food available to it is proportional to the area of its territory, so a larger territory provides more food.

On the other hand a larger territory has a larger perimeter to defend from intruders. If more time has to be spent defending the larger territory then less time is available to harvest the available

food. In the worst case all the time might have to be spent defending the perimeter leaving no time for harvesting. On the other hand if the territory is too small then it may not contain enough food to support the individual. There seems to be a tradeoff here, and that suggests that there is

some optimal size for the territory that represents the best tradeoff. If the bird's territory is circular in shape what radius of territory will maximize the food harvested?

$$F \propto A = \pi r^2 \Rightarrow F = k_1 \pi r^2$$

$$H = t_H \cdot F = t_H \cdot k_1 \pi r^2$$

$$= (1 - t_D) k_1 \pi r^2$$

$$t_D + t_H = 1$$

$$t_H = 1 - t_D$$

$$H = (1 - 2k_2 \pi r) k_1 \pi r^2$$

$$H = f(r)$$

$$t_D = k_2 2\pi r$$

$$H = k_1 \pi r^2 - 2k_1 k_2 \pi^2 r^3$$

$$\frac{dH}{dr} = \pi \cdot 2r - 2k_2 \pi r^2 \cdot 3r^2 \stackrel{!}{=} 0 \quad \leftarrow$$

$$r - 3k_2 \pi r^2 \stackrel{!}{=} 0 \quad \leftarrow$$

$$r(1 - 3k_2 \pi r) \stackrel{!}{=} 0$$

$r=0$

$$1 - 3k_2 \pi r = 0$$

$$r = \frac{1}{3k_2 \pi}$$

