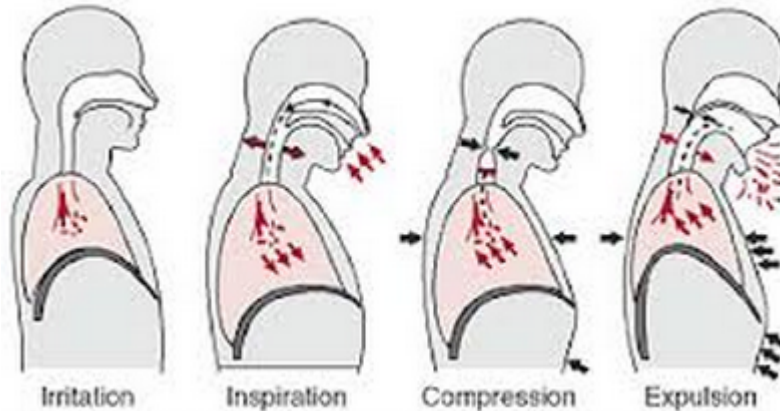


Cough-cough



What is the optimum radius of the trachea when coughing? The goal is to create a strong outward force in the throat to clear it.

For this problem you should perform the following experiment:

Hold your hand about 10 cm from your mouth and blow on it (i) with your lips compressed almost closed but with a small stream of air escaping, (ii) with your mouth wide open, and (iii) with your lips adjusted to create the largest force on your hand.

With (i) your lips almost closed there is a high pressure causing rapid air flow but a small stream of air and little force. With (ii) your mouth wide open there is a large stream of air but with little pressure so that air flow is slow. The largest force (iii) is created with an intermediate opening of your lips where there is a notable pressure and rapid flow of substantial volume of air.

Let R be the normal radius of the trachea and r be the reduced radius when coughing. Assume that the velocity of air flow through the trachea is proportional to pressure difference across the trachea and that the pressure difference is proportional to the constriction of the trachea. Assume that the mass of air flow is proportional to the area of the trachea, and that its cross-section is circular. Finally the momentum, M , of the air flow is the product of the mass of the air flow and its velocity.

We would like to find the value of r that maximizes M .

a) If R is the normal radius of the trachea and r is the reduced radius write an expression for the constriction.

$$R - r$$

b) Use your expression from a) to translate the sentence “Assume that the velocity of air flow through the trachea is proportional to pressure difference across the trachea and that the pressure difference is proportional to the constriction of the trachea.” into an equation for the velocity v of the air flow.

$$v \propto \Delta P \propto R - r$$

$$v = k_1 (R - r)$$

c) What is an expression for the area of a circle?

$$\pi r^2 = A$$

d) Use your expression from c) to translate the sentence "Assume that the mass of air flow is proportional to the area of the trachea, and that its cross-section is circular." Into an equation for, m , the mass of the air flow.

$$m \propto A = \pi r^2$$

$$m = k_2 \pi r^2$$

e) Combine your equations from b) and c) to translate the sentence "Finally the momentum, M , of the air flow is the product of the mass of the air flow and its velocity." into an equation for M in terms of r .

$$M = m \times v$$

$$= k_2 \pi r^2 (k_1 (R - r))$$

f) Find the value of r that maximizes M by taking the derivative of M with respect to r setting it equal to 0 and solving for r .

$$\frac{dM}{dr} = \dots$$

g) Since we don't have an exact numerical value for R our answer in f) will be in terms of R . Interpret that expression.

$$M = k_1 k_2 \pi r^2 R - k_1 k_2 \pi r^3$$

$$\frac{dM}{dr} = k_1 k_2 \pi R \cdot 2r - k_1 k_2 \pi \cdot 3r^2 \stackrel{!}{=} 0$$

$$2Rr - 3r^2 \stackrel{!}{=} 0$$

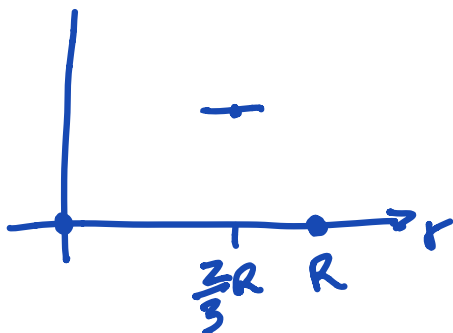
$$r(2R - 3r) \stackrel{!}{=} 0$$

$$2R - 3r = 0$$

$r=0$

$$r = \frac{+2R}{+3} = \frac{2}{3} R$$

constriction of $\frac{1}{3}$, 33%



$$@ r=R$$

$$M = k_2 \pi r^2 (k_1 (R - r))$$

$$= k_2 \pi R^2 (k_1 (R - R))$$

$$= 0$$