

Exercise set 8.3: Differential Equations

Solving differential equations can be difficult, but testing proposed solutions is easier: we can simply substitute the proposed solution into the differential equation and see if it satisfies it. For example to see if $y = \cos t$ is a¹ solution to $y'' + y = 0$ we calculate y'' ,

$$y = \cos t \Rightarrow y' = -\sin t \Rightarrow y'' = -\cos t$$

and substitute,

$$y'' + y = 0 \Rightarrow -\cos t + \cos t = 0$$

Since this equation is true, $y = \cos t$ is in fact a solution to the differential equation $y'' + y = 0$.

Test the following proposed solutions to see which ones satisfy the associated equations.

1. Equation: $y' - 2 = 0$. Proposed solution: $y = 2x$.
2. Equation: $y' - 2 = 0$. Proposed solution: $y = 2x + 1$.
3. Equation: $y' - 2 = 0$. Proposed solution: $y = 2x + p$.

(What conclusion can you draw from the first three problems?)

4. Equation: $y' - y = 0$. Proposed solution: $y = x^2$.
5. Equation: $y' - y = 0$. Proposed solution: $y = e^x$.
6. Equation: $y'' + y = 0$. Proposed solution: $y = x^2$.
7. Equation: $y'' + y - 2 = 0$. Proposed solution: $y = x^2$.
8. Equation: $y'' + 4y = 0$. Proposed solution: $y = \cos 4x$.
9. Equation: $y'' + y = 0$. Proposed solution: $y = \sin t + \cos t$.
10. Equation: $y'' + y = 0$. Proposed solution: $y = 3 \sin t + 2 \cos t$.
11. Equation: $y'' + 2y' + y = 0$. Proposed solution: $y = e^{-t}$.
12. Equation: $y'' + 2y' + 2y = 0$. Proposed solution: $y = e^{-t} \sin t$.

¹ It may not be the **only** solution.