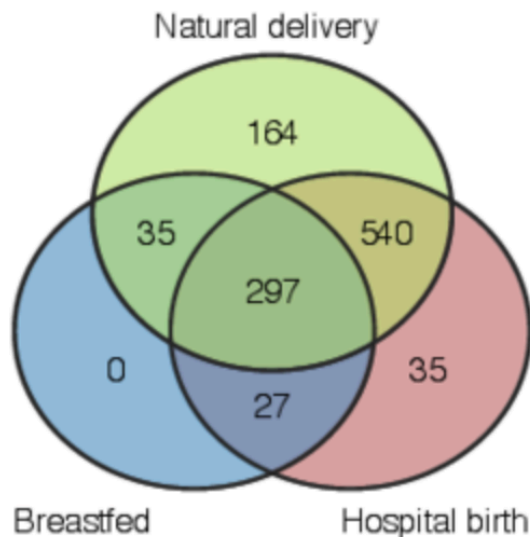


1. (5 marks) **A subtle family challenge.** All the Parker daughters, Alice, Betty and Carol, are avid tennis players. Betty is the best player while Carol and Alice are evenly matched: Alice defeats Carol 50% of the time, but only defeats Betty 40% of the time.

Alice's mother offers to give her \$100 if she plays three games against her sisters and wins two in a row. Alice quickly suggests she will play Carol three times, but her mother just as quickly says she has to alternate opponents. Alice wants the \$100, but isn't sure in what sequence to face her sisters.

- Use a tree diagram to calculate the probability of Alice winning the \$100 if she chooses to play Betty, then Carol, then Betty again (BCB).
 - Use a tree diagram to calculate the probability of Alice winning the \$100 if she chooses to play Carol, then Betty, then Carol again (CBC).
 - Should she play Betty then Carol then Betty again (BCB), or play Carol, then Betty then Carol again (CBC)?
 - How would you explain to someone who didn't know probability your answer in c. above?
2. (10 marks) **More playoffs.** Continuing to work with teams A and B who played four times in the regular season with team winning 3 times and team B once.
- In the pencast "Example: The playoffs" I kept the probabilities of A beating B constant through the playoffs (always 0.75), but in class we updated them as the series unfolded based on the results as teams won and lost. For instance if A wins the first game, it has now won 4 of 5 games against B and its probability of winning the next game would seem to be 0.8, not 0.75. Similarly if B wins the first game, it will have won 2 of 5 games against A and its probability of winning the next game could go up to 0.4 from 0.25. But does doing this make a difference, i.e. does the probability of team B winning a best-of-three series change?
 - There's an old saying in sports that "Longer series favour the stronger team." What are team B's chances of winning a single elimination game with A? A best of three series? A best of 5 series? Does the saying seem to be true?
 - In a best of 5 series what is the probability that the series will be tied at some point?
 - In a best of 5 series what is the probability that the winning team "came from behind", i.e. was behind at some point in the series?

3. (5 marks) **Birth.** As part of an analysis of gut microbiome variation Dutch researchers collected data on early-life factors. They distinguished between people born in a hospital vs born at home, between natural delivery and caesarian section, and between being breastfed or not. Here's a summary of their counts across these categories:



- What is the probability that someone in their dataset was born in a hospital?
 - Did a hospital birth make one more or less likely to be breastfed?
 - What combination of factors made one most likely to be breastfed?
 - What is the interpretation of the 0 entry in the diagram?
4. (10 marks) **Our relatives.** Chimpanzees and bonobos are humans' closest relatives among the great apes, and both have genomes that overlap a great deal with ours.
- “Bonobos and chimpanzees are highly similar to each other in many respects. However, the behaviour of the two species differs in important ways. For example, male chimpanzees use aggression to compete for dominance rank and obtain sex, and they cooperate to defend their home range and attack other groups. By contrast, bonobo males are commonly subordinate to females and do not compete intensely for dominance rank. They do not form alliances with one another and there is no evidence of lethal aggression between groups. Compared with chimpanzees, bonobos are playful throughout their lives and show intense sexual behaviour that serves non-conceptive functions and often

involves same-sex partners. Thus, chimpanzees and bonobos each possess certain characteristics that are more similar to human traits than they are to one another's.”¹

Genomes can be compared according to the amount of shared genomic material measured in Megabases, Mb². Human and chimpanzee genomes share around 69 Mb of genome segments, chimpanzees and bonobos around 85.4 Mb, and bonobos and humans around 68.7 Mb. Of their shared segments chimpanzees and bonobos have 17.2 Mb that are not shared with humans. In total chimpanzees have slightly more genomic segments than bonobos, 87 Mb vs 86.6 Mb. Across the three species there are 102.8 Mb unique segments.

- a. What is the probability that a segment is common across all three species?
 - b. Given a segment is shared, which pair of species is it most likely to be shared between? With what probability?
 - c. Given that a segment is shared between humans and another species, which other species is it more likely to be? With what probability?
5. (10 marks) **Bird Nests.** In a study of bird nesting habits Martin, Aitken and Wiebe³ reported the following observations about their study plot:
- It contained 10,719 available trees, 690 of which contained at least one nest.
 - It contained four tree species: 1,562 Quaking aspen, 3,260 Douglas-fir, 4,296 Lodgepole pine, and 1,601 Hybrid spruce.
 - There were nests in 658 of the Quaking aspen, 9 of the Douglas-fir, 18 of the Lodgepole pine, and 6 of the Hybrid spruce.
 - 1,612 nests were observed in Quaking aspen trees, 19 in Douglas-fir, 48 in Lodgepole pine and 10 in Hybrid spruce.
 - In one subsample of trees with nests 87 trees had a single nest, 51 of which were in use that season, 99 trees had 2 or 3 nests, in 33 of which at least one nest was in use, and 33 trees had more than 3 nests, only 5 of which had nests in use that season.
- a. If a tree is selected at random what is the probability that it is a Quaking aspen? That it is a Douglas-fir?
 - b. If a tree is selected at random what is the probability that it has at least one nest in it?
 - c. If a tree is a Quaking aspen what is the probability that it has at least one nest in it? What if it is a Douglas-fir?

¹ The bonobo genome compared with the chimpanzee and human genomes. Kay Prüfer et. al. Nature 486, 527–531(2012).

² Megabase: unit of length for DNA fragments equal to 1 million nucleotides.

³ Martin, K., K.E.H. Aitken and K.L. Wiebe. 2004. Nest sites and nest webs for cavity-nesting communities in interior British Columbia, Canada: nest characteristics and niche partitioning. Condor 106: 5-19

- d. If a tree has a nest in it what is the probability that it is a Quaking aspen? That it is a Douglas-fir?
 - e. If you spot a nest in a tree what is the probability that it is the only nest in the tree?
 - f. If you spot a nest in a tree what is the probability that it has been in use this season?
6. (20 marks) **Craps.** Casinos are laboratories for probability and foremost among the games of chance there is the game of Craps. Sam Smith of Saint Joseph's University explains why:

Played on a long green velvet table with one roller and many side betters and spectators, "Craps" is the classic casino dice game. The game can go on indefinitely, in principle, with the tension and side bets mounting with each throw. The rules are simple: The roller throws the dice. If she rolls a sum of 7 or 11 she wins. If she throws a sum of 2, 3 or 12 ("craps") she loses. Otherwise call the sum she rolls the *point P*. The roller continually rolls until either (i) she rolls the sum *P* again or (ii) she rolls a 7. In the first case the roller wins the bet and in the second she loses. Spectators may bet with the roller or on a number of side bets listed on the table.

If you'd like some atmosphere before beginning the problem you can watch this:

http://www.youtube.com/watch?v=HQiYT_AdGdU

- a. What is the probability that the roller wins on her first roll, $p(\text{Winning on 1}^{\text{st}} \text{ roll})$?
- b. What is the probability that the roller loses on her first roll, $p(\text{Losing on 1}^{\text{st}} \text{ roll})$?
- c. If the roller's first roll is a 4 she neither wins nor loses immediately, but instead must keep rolling until either she rolls another 4 and wins, or she rolls a 7 and loses. What is the probability that she rolls a 4? What is the probability that she rolls a 7? What is the probability that she rolls a 4 before a 7, i.e. what is the probability that she wins, $p(\text{Winning} \mid 4 \text{ on } 1^{\text{st}} \text{ roll})$?
- d. The last probability in c. is the probability of winning given that she rolls a 4 on the first roll, $p(\text{Winning} \mid 4 \text{ on } 1^{\text{st}} \text{ roll})$. Find the equivalent probabilities for first rolls of 5, 6, 8, 9, 10.
- e. You now have probabilities of winning for every first roll from 2 to 12. Combine these to arrive at the probability of winning a game of Craps.
- f. In his description Sam Smith says "The game can go on indefinitely, in principle..." Let's investigate why he says "in principle". Assuming the roller's first roll is a 4, what is the probability that the game ends in one more roll, i.e. on the next roll? That it ends within 2 more rolls? Within 5 more rolls? 10 more rolls? 100 more rolls? 1,000 more rolls?