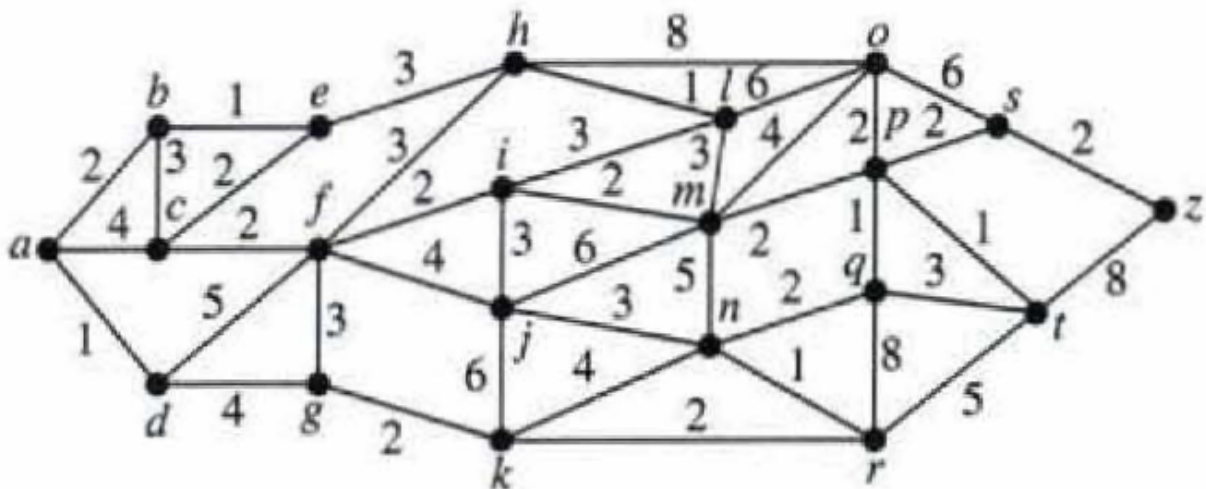


1. (15 marks) Consider the graph represented by this matrix:

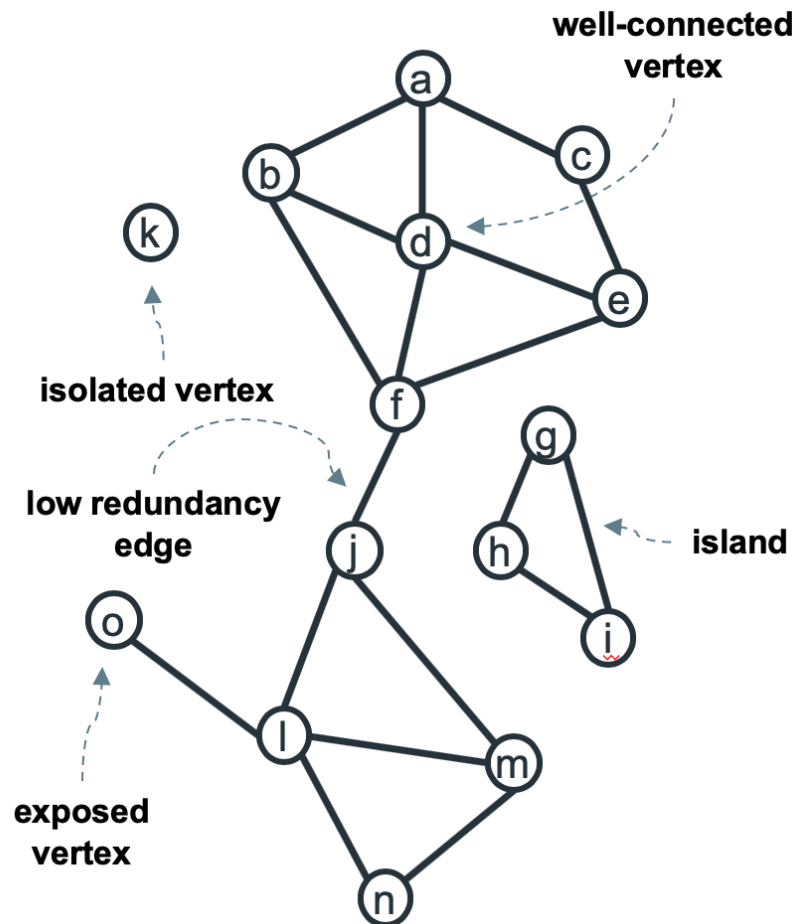
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
<i>a</i>	—	3	1	—	5
<i>b</i>	3	—	2	7	6
<i>c</i>	1	2	—	8	—
<i>d</i>	—	7	8	4	9
<i>e</i>	5	6	—	9	—

- How many vertices does it have?
 - How many edges does it have?
 - Is it directed or undirected?
 - Is it weighted or unweighted?
 - Does it contain any loops? (An edge connecting a vertex to itself.)
 - Does it have an Euler circuit? An Euler path?
 - Which edges make up the minimal spanning tree of the graph?
 - Draw the visual representation of the graph with vertices and edges.
2. (10 marks) What is the shortest route from *a* to *z* in the graph below? How long is it? (You can do this on-the-graph or in-the-matrix, whichever you prefer, but be sure to show your work. I want to be able to see that you applied the algorithm properly.)



3. (15 marks) *Habitat connectivity*

Graphs can make clear a number of habitat features of interest, e.g. the labelled items in the graph below. These are easy to spot in a small modestly connected visual graph, but how to spot them in a matrix representation of a graph?

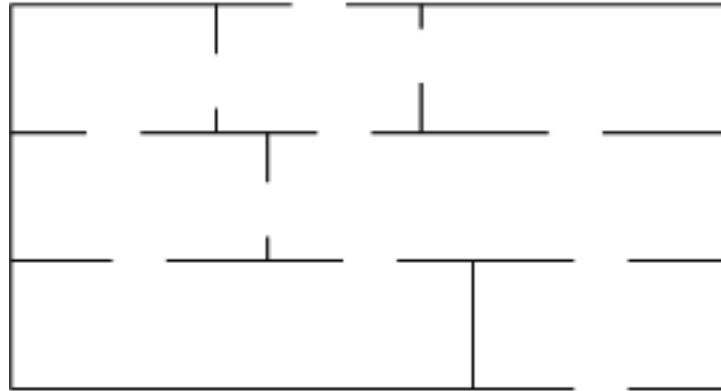


a. Draw the matrix representation of this graph.

Informed by this example write instructions telling someone how to identify the presence of each of the following features of a graph in the matrix, i.e. by looking only at the matrix and not consulting the picture of the graph.

- The best-connected vertex in the graph.
- Isolated vertices.
- Low redundancy edges.
- Exposed vertices.
- Islands.

4. (5 marks) The diagram below shows the floor plan of a house. Is it possible to walk through the house and go through every doorway exactly once (including the front and back doors)? If it is, show the path. If it isn't, explain why not.



5. (10 marks) *Pitcher problems and linear equations*

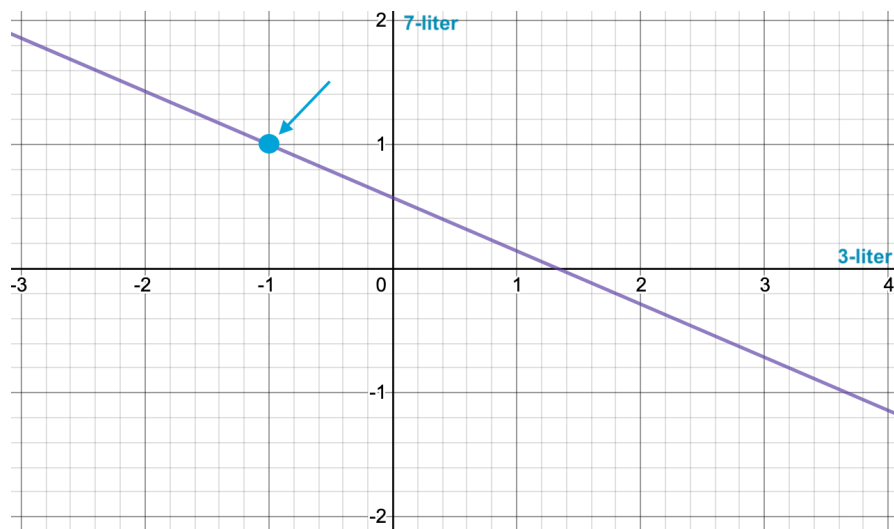
There is more than one way to solve most problems and pitcher problems are no exception. For example the problem of measuring volumes of liquid using a 3-liter and a 7-liter pitcher can be formulated as an algebraic problem, specifically by the expression

$$3x + 7y$$

where x represents the number of times the 3-liter pitcher is filled, and y represents the number of times we fill the 7-liter pitcher. A positive value means the pitcher is filled while a negative value means it is emptied. For example $x = +1$ and $y = +1$ corresponds to filling both pitchers, and making 10 gallons. Here's how to use the expression. Let's say we want to measure 4 liters. We plot the equation,

$$3x + 7y = 4$$

And look for places the line crosses the intersections of integer coordinates. There is only one in this case at $(-1, 1)$.



What does this mean? Well $x = -1$ and $y = +1$ so it means we should empty the 3-liter pitcher once (-1) and fill the 7-liter pitcher once $(+1)$. What actions does that imply? After all we can't start by emptying the 3-liter pitcher because there's nothing in it to start with. It means we should fill the 7-liter pitcher, pour from it into the 3-liter pitcher, and then empty the 3-liter pitcher.

- a. By setting the expression $3x + 7y$ to every value from 1 through 10 and plotting the equations that result, verify which amounts can be measured with these two pitchers, and write out the filling and pouring instructions for each amount.

Here are some more examples of lines with their integer intersections marked by circles. (Why only integer intersections? Because we can't partly fill or empty the pitchers.)

