

Exercise set 13: Matrix Multiplication

1. Calculate the entries for the three boxes in the matrix multiplication below:

$$\begin{bmatrix} 1 & 7 & 9 & 3 \\ 5 & 6 & 6 & 2 \\ 2 & 1 & 3 & 4 \\ 8 & 3 & 8 & 1 \end{bmatrix} \begin{bmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{bmatrix} = \begin{bmatrix} 3 & 4 & 9 & 7 \\ 4 & 8 & 5 & 6 \\ 1 & 2 & 2 & 4 \\ 7 & 3 & 2 & 9 \end{bmatrix}$$

$= 3(1) + 4(7) + 1(9) + 7(3) = 61$
 $= 9(5) + 5(6) + 2(6) + 2(2) = 91$
 $= 4(8) + 8(3) + 2(8) + 3(1) = 75$

2. Find A^2 , A^3 , A^4 and A^5 for each of the following,

a. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Since $A^2 = A$ so with A^3, A^4, A^5 .
 This pattern of matrix with 1s on the main diagonal is called the identity matrix.

b. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ A "rotating" matrix
 $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A^3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A^5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, ...
 $A^2, A^4, A^6, \dots$ and $A, A^3, A^5, A^7, \dots$

c. $A = \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix} \xrightarrow{A^2} \begin{bmatrix} 0.5 & 0.5 \\ 0.25 & 0.75 \end{bmatrix} \xrightarrow{A^3} \begin{bmatrix} 0.375 & 0.625 \\ 0.3125 & 0.6875 \end{bmatrix} \xrightarrow{A^4} \begin{bmatrix} 0.3125 & 0.6875 \\ 0.34375 & 0.65625 \end{bmatrix} \xrightarrow{A^5} \begin{bmatrix} 0.3125 & 0.6875 \\ 0.34375 & 0.65625 \end{bmatrix}$$

d. $A = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$

$$\begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \xrightarrow{A^2} \begin{bmatrix} 0.44 & 0.56 \\ 0.28 & 0.72 \end{bmatrix} \xrightarrow{A^3} \begin{bmatrix} 0.376 & 0.624 \\ 0.312 & 0.688 \end{bmatrix} \xrightarrow{A^4} \begin{bmatrix} 0.3504 & 0.6496 \\ 0.3248 & 0.6752 \end{bmatrix} \xrightarrow{A^5} \begin{bmatrix} 0.34016 & 0.65984 \\ 0.32992 & 0.67008 \end{bmatrix}$$