

Exercise set 13.2: Election Theory

1. Suppose plurality is being used to decide an election. Compare the minimal percentage of the vote necessary to win the election if there are:

a. Two candidates $50\% + 1$

b. Three candidates $33.\bar{3}\% + 1$

c. n candidates $\frac{100}{n}\% + 1$

2. If there are four candidates in an election, how many different possible preference schedules are there (assuming ties are not allowed)? $4 \cdot 3 \cdot 2 \cdot 1 = 4!$

3. Determine winners by each method for this set of preference schedules:

Plurality		<u>A</u>		<u>B</u>		<u>C</u>		<u>D</u>	
		11		10		5		4	
$n = \# \text{ ballots}$		5	6	5	5	4	1	4	
Most preferred		A	A	B	B	C	C	D	
		C	D	C	D	D	B	C	
		D	C	D	C	B	A	B	
Least preferred		B	B	A	A	A	D	A	

A wins with 11 votes

Majority

$n = \# \text{ ballots}$	5	6	5	5	4	1	4	$N = 5+6+5+5+4+1+4$
Most preferred	A	A	B	B	~~C~~	~~C~~	~~D~~	$4+1+4$
	C	D	C	D	~~D~~	B	~~C~~	$= 30$
	D	C	D	C	B	A	B	
Least preferred	B	B	A	A	A	D	A	$\therefore 16 \text{ votes to win}$

		Votes			
Round	A	B	C	D	
1	$5+6=11$	$5+5=10$	$4+1=5$	4	
2	"	"	$4+1+4=9$	—	
3	$5+6=11$	$5+5+4+1+4=19$			$\therefore B \text{ wins}$

Borda count (low is good)

$n = \#$ ballots	5	6	5	5	4	1	4	
Most preferred	A	A	B	B	C	C	D	1
	C	D	C	D	D	B	C	2
	D	C	D	C	B	A	B	3
Least preferred	B	B	A	A	A	D	A	4

$$A : 5 \times 1 + 6 \times 1 + 5 \times 4 + 5 \times 4 + 4 \times 4 + 1 \times 3 + 4 \times 4 = 86$$

$$B : 5 \times 4 + 6 \times 4 + 5 \times 1 + 5 \times 1 + 4 \times 3 + 1 \times 2 + 4 \times 3 = 80$$

$$C : 5 \times 2 + 6 \times 3 + 5 \times 2 + 5 \times 3 + 4 \times 1 + 1 \times 1 + 4 \times 2 = 66$$

$$D : 5 \times 3 + 6 \times 2 + 5 \times 3 + 5 \times 2 + 4 \times 2 + 1 \times 4 + 4 \times 1 = 68$$

C is Borda winner

Condorcet

$n = \# \text{ ballots}$	5	6	5	5	4	1	4
Most preferred	A	A	B	B	C	C	D
	C	D	C	D	D	B	C
	D	C	D	C	B	A	B
Least preferred	B	B	A	A	A	D	A

No clear intuition so start with A vs B:

<u>A vs B</u>		→ B wins vs A so try B vs C:	
5			
6			
	5		
	5		
	4		
	1		
	4		
<u>11</u>	<u>19</u>		
		<u>B C</u>	
		5	
		6	
		5	
		5	
		4	
		1	
		4	
		<u>10</u>	<u>20</u>
		<u>C D</u>	
		5	
			6
		5	
			5
		4	
		1	
			4
		<u>15</u>	<u>15</u>
		A tie !?	

Let's try C & D against the others:

$$\begin{array}{r}
 \text{C vs A} \\
 \hline
 \begin{array}{r}
 5 \\
 5 \\
 4 \\
 1 \\
 4 \\
 \hline
 19
 \end{array}
 \begin{array}{r}
 5 \\
 6 \\
 \hline
 11
 \end{array}
 \end{array}$$

So C beats A + B
and ties D

Now D vs A and B:

$$\begin{array}{r}
 \text{D A} \\
 \hline
 \begin{array}{r}
 5 \\
 5 \\
 4 \\
 4 \\
 \hline
 18
 \end{array}
 \begin{array}{r}
 5 \\
 6 \\
 1 \\
 \hline
 12
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{D B} \\
 \hline
 \begin{array}{r}
 5 \\
 6 \\
 4 \\
 4 \\
 \hline
 19
 \end{array}
 \begin{array}{r}
 5 \\
 5 \\
 1 \\
 \hline
 11
 \end{array}
 \end{array}$$

D beats A + B

So C + D are truly a Condorcet tie

$\therefore$ they are each preferred to A + B
and tied with each other.