

Exercise set 2.1

1. Write the matrices corresponding to these systems of equations: ** just keep the numbers **

$$\begin{array}{rcl} 2x + y - z & = & 8 \\ -x + 3y + 5z & = & 1 \\ 4x - 2y + 2z & = & 11 \end{array} \quad \begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ -1 & 3 & 5 & 1 \\ 4 & -2 & 2 & 11 \end{array} \right] \end{array}$$

$$\begin{array}{rcl} 1.53x + 0.004y & = & 5.71 \\ 0.083x - 0.052y & = & 1.028 \end{array} \quad \begin{array}{c} x \quad y \\ \left[\begin{array}{cc|c} 1.53 & 0.004 & 5.71 \\ 0.083 & -0.052 & 1.028 \end{array} \right] \end{array}$$

Missing terms are like 0 coefficients:

$$\begin{array}{rcl} 3x + y & = & 5 \\ 2y + 3z & = & 12 \\ x + 5z & = & -3 \end{array} \quad \left. \begin{array}{l} 3x + y + 0z = 5 \\ 0x + 2y + 3z = 12 \\ x + 0y + 5z = -3 \end{array} \right\} \quad \begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} 3 & 1 & 0 & 5 \\ 0 & 2 & 3 & 12 \\ 1 & 0 & 5 & -3 \end{array} \right] \end{array}$$

Need to tidy up first:

$$\begin{array}{rcl} 2x + y & = & 8 + z \\ 5z - x - 1 & = & -3y \\ 4x - 11 & = & 2y - 2z \end{array} \quad \left. \begin{array}{l} 2x + y - z = 8 \\ -x + 3y + 5z = 1 \\ 4x - 2y + 2z = 11 \end{array} \right\} \quad \begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ -1 & 3 & 5 & 1 \\ 4 & -2 & 2 & 11 \end{array} \right] \end{array}$$

2. And now write the equations corresponding to these matrices:

$$\begin{array}{c} x \quad y \\ \left[\begin{array}{cc|c} 1 & 1 & 7 \\ 1 & -1 & 3 \end{array} \right] \end{array} \quad \left. \begin{array}{l} 1x + 1y = 7 \\ 1x - 1y = 3 \end{array} \right\} \quad \begin{array}{l} x + y = 7 \\ x - y = 3 \end{array}$$

$$\begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} 1 & -1 & 2 & 7 \\ 3 & 0 & 5 & -12 \\ -2 & 3 & 1 & 8 \end{array} \right] \end{array} \quad \left. \begin{array}{l} x - y + 2z = 7 \\ 3x + 5z = -12 \\ -2x + 3y + z = 8 \end{array} \right\}$$

$$\begin{array}{c} a \quad b \quad c \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1.3 \\ 0 & 1 & 0 & -0.8 \\ 0 & 0 & 1 & 2.1 \end{array} \right] \end{array} \quad \left. \begin{array}{l} a = 1.3 \\ b = -0.8 \\ c = 2.1 \end{array} \right\}$$

3. Perform the following operations on the matrix below:

- Divide the first row by 2: $R1 \div 2 \rightarrow R1$
- Divide the second row by 3: $R2 \div 3 \rightarrow R2$
- Multiply the last row by 0.5: $R3 \times 0.5 \rightarrow R3$

$$\begin{bmatrix} 6 & -1 & 2 & 7 \\ 3 & 0 & 5 & -12 \\ 6 & -1 & 2 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{6}{2} & \frac{-1}{2} & \frac{2}{2} & \frac{7}{2} \\ \frac{3}{3} & \frac{0}{3} & \frac{5}{3} & \frac{-12}{3} \\ \frac{6}{2} & \frac{-1}{2} & \frac{2}{2} & \frac{7}{2} \end{bmatrix} = \begin{bmatrix} 3 & -1/2 & 1 & 7/2 \\ 1 & 0 & 5/3 & -4 \\ 3 & -1/2 & 1 & 7/2 \end{bmatrix}$$

4. Perform the following operations on the matrices below:

Add twice the first row to the second: $2 \times R1 + R2 \rightarrow R2$

$$\begin{bmatrix} 1 & 1 & 7 \\ 1 & -1 & 3 \end{bmatrix} \xrightarrow{2R_1 = 2 \cdot 1 \quad 2 \cdot 1 \quad 2 \cdot 7 = 2 \quad 2 \quad 14} \begin{array}{r} 2R_1 = 2 \cdot 1 \quad 2 \cdot 1 \quad 2 \cdot 7 = 2 \quad 2 \quad 14 \\ + R_2 = 1 \quad -1 \quad 3 \\ \hline 2R_1 + R_2 = 3 \quad 1 \quad 17 \end{array}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 7 \\ 3 & 1 & 17 \end{bmatrix}$$

Add three times the 3rd row to the 1st: $3 \times R3 + R1 \rightarrow R1$ ①

and subtract the third from the second: $-1 \times R3 + R2 \rightarrow R2$ ②

$$\begin{bmatrix} 1 & 0 & 0 & 1.3 \\ 0 & 1 & 0 & -0.8 \\ 0 & 0 & 1 & 2.1 \end{bmatrix} \xrightarrow{\text{① } 3R_3 = 0 \quad 0 \quad 3 \quad 6.3} \begin{array}{r} 3R_3 = 0 \quad 0 \quad 3 \quad 6.3 \\ + R_1 = 1 \quad 0 \quad 0 \quad 1.3 \\ \hline 3R_3 + R_1 = 1 \quad 0 \quad 3 \quad 7.6 \end{array}$$

$$\xrightarrow{\text{② } -R_3 = 0 \quad 0 \quad -1 \quad -2.1} \begin{array}{r} -R_3 = 0 \quad 0 \quad -1 \quad -2.1 \\ + R_2 = 0 \quad 1 \quad 0 \quad -0.8 \\ \hline -R_3 + R_2 = 0 \quad 1 \quad -1 \quad -2.9 \end{array}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 3 & 7.6 \\ 0 & 1 & -1 & -2.9 \\ 0 & 0 & 1 & 2.1 \end{bmatrix}$$

Add three times the 3rd row to the 1st: $3 \times R3 + R1 \rightarrow R1$ ①

and subtract the third from the second: $-1 \times R3 + R2 \rightarrow R2$ ②

$$\begin{bmatrix} 1 & -1 & 2 & 7 \\ 3 & 0 & 5 & -12 \\ -2 & 3 & 1 & 8 \end{bmatrix} \xrightarrow{\text{① } 3R_3 = -6 \quad 9 \quad 3 \quad 24} \begin{array}{r} 3R_3 = -6 \quad 9 \quad 3 \quad 24 \\ + R_1 = 1 \quad -1 \quad 2 \quad 7 \\ \hline 3R_3 + R_1 = -5 \quad 8 \quad 5 \quad 31 \end{array}$$

$$\xrightarrow{\text{② } -R_3 = 2 \quad -3 \quad -1 \quad -8} \begin{array}{r} -R_3 = 2 \quad -3 \quad -1 \quad -8 \\ + R_2 = 3 \quad 0 \quad 5 \quad -12 \\ \hline -R_3 + R_2 = 5 \quad -3 \quad 4 \quad -20 \end{array}$$

$$\rightarrow \begin{bmatrix} -5 & 8 & 5 & 31 \\ 5 & -3 & 4 & -20 \\ -2 & 3 & 1 & 8 \end{bmatrix}$$