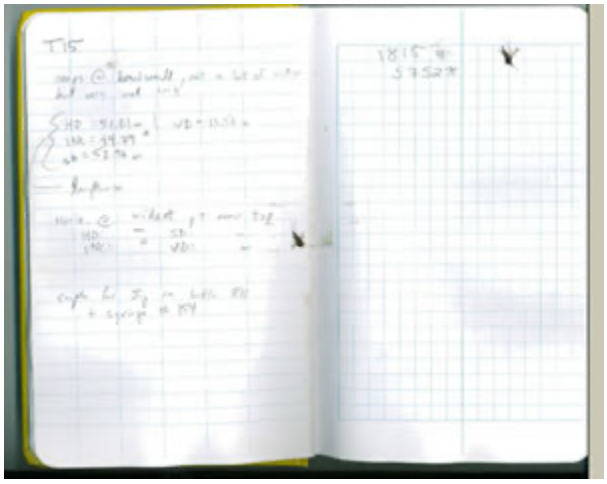
ALES 291/Math 120 **Assignment 2** Due: 1 pm Jan 19, 2024

1. (*5 marks*) The perils of an incomplete notebook

You are working with an entomologist and are trying to repeat a previous experiment under a different set of conditions. The lab notebook (note the dead mosquitoes from field season!) indicates that the first trial began with a total population of 200 insects made up of a mix of adults and juveniles, and then records that the experimenter removed 11 adults from that culture so the juvenile to adult ratio would be 3:4, but it doesn't record how many of the original 200 were adults and how many were juveniles.



2. (*10 marks*) Bundles of grain

The oldest known simultaneous equations solved using something we would recognize as matrix methods are these (from the Chinese document "The nine chapters on the mathematical art" circa 200 BCE). The first problem of Chapter 8 is:

> 3 bundles of high-grade grain, 2 bundles of medium-grade grain and 1 bundle of low-grade grain give 39 units. 2 bundles of high-grade grain, 3 bundles of medium-grade grain, and 1 bundle of low-grade grain give 34 units. 1 bundle of high-grade grain, 2 bundles of medium-grade grain, and 3 bundles of low-grade grain yields 26 units. How many units does one bundle of each grain yield?

3. (*10 marks*) Forensic accounting

Three groups of people enter a museum. The first group of two adults, two youth and one child pays \$22.50 admission. The second group consisting of one adult, one youth and three children pays \$17.50. The third group of three adults, one youth, and two children pays \$27.00. How much is the individual admission for an adult? For a youth? For a child?

4. (*10 marks*) Three systems, two degenerate

What is the solution to the systems of equations represented by each of these matrices? (Assume the variables are x, y and z.)

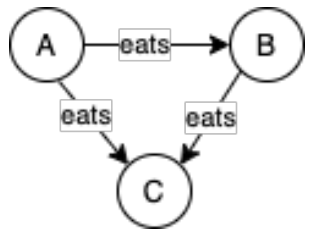
a) $\begin{bmatrix} 3 & -6 & 9 & 0 \\ -4 & -6 & 8 & -4 \\ -2 & -1 & 1 & 7 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 2 & 4 & 8 \\ 1 & -1 & 2 & 2 \\ -1 & 5 & -2 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & -1 & 1 & 3 \\ 1 & 1 & -1 & 5 \\ -2 & 4 & -4 & 1 \end{bmatrix}$

5. (*10 marks*) Another use of matrices

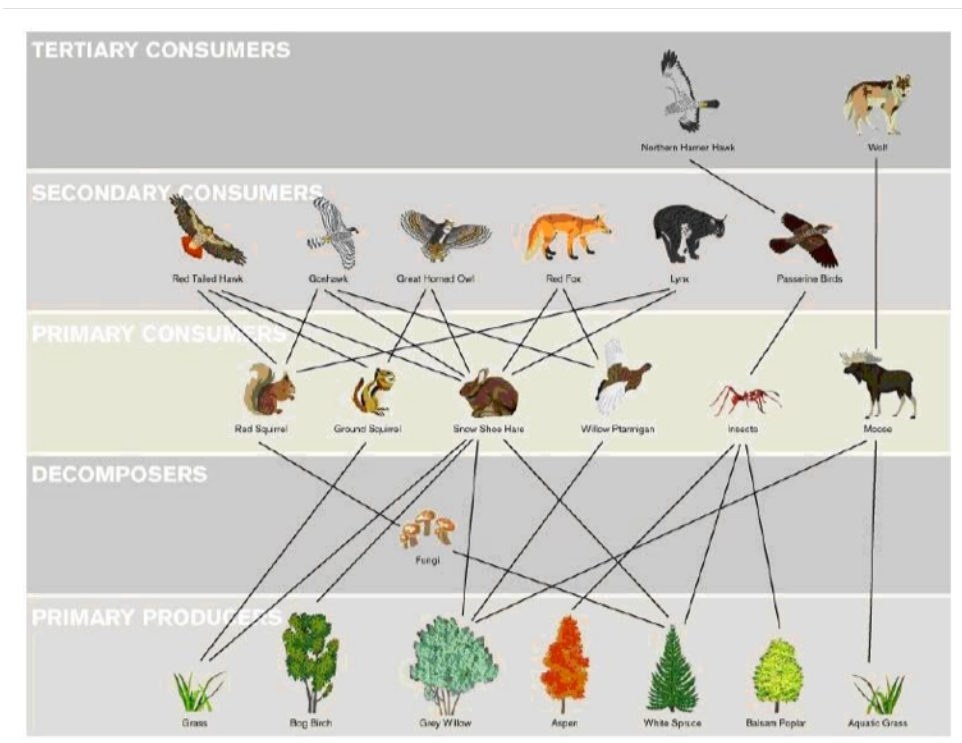
As you saw in the slide show matrices can be used for many things. Here is a use quite different than representing systems of linear equations: representing food webs. A food web is a who-eats-who map of an ecosystem. A very simple graphical food web might look like this,



Where As and Bs both eat Cs, and As also eat Bs. We could represent these relations using a matrix of 1s and 0s where a 1 at the entry in row *r* and column *c* means that *r*'s eat *c*'s, and a 0 means they don't. For example the 1 at the intersection of row A and column B means that As eats Bs.

$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Here's a somewhat larger food web for the Kluane region:



a) Represent this food web using a matrix (yes it will be fairly large).

In case you are having trouble identifying the species here's what they look like to me (left to right, top to bottom, name followed by initials to use for row and column labelling): Northern Harrier Hawk NHH, Wolf W, Red Tailed Hawk RTH, Goshawk GH, Great Horned Owl O, Red Fox F, Lynx L, Passerine Birds PB, Red Squirrel RS, Ground Squirrel GS, Snow Shoe Hare H, Willow Ptarmigan P, Insects I, Moose M, Fungi FU, Grass G, Bog Birch BB, Grey Willow GW, Aspen A, White Spruce WS, Balsam Poplar BP, Aquatic Grass AG.

b) How do the numbers of 0s and 1s compare? What does the imbalance tell us about the food web?

c) Find the totals of each row.

d) What does the row with the largest total represent?

e) Are there any rows that are all 0s? What do these represent?

f) What is the interpretation of a row with a single 1 in it?

g) Find the totals of each column.

h) What does the column with largest total represent?

i) Are there any columns that are all 0s? What do these represent?

6. (*5 marks*) Is the sum of two odd numbers always even? How would you explain to someone why or why not?

This problem is an exercise in communication, not mathematical manipulations. Your explanation can be any combination of words, pictures, and numerical examples. Strive to make it as simple, clear, convincing, and short as you can. The ideal answer would be a single phrase or simple diagram, though this may not be attainable.