

1. (10 marks) Fitness regimes

You can do almost everything online these days, including getting a fitness consultation. The better sites lead you through a virtual interview where you fill in web forms to identify your fitness goals, the forms of exercise available to you, your preferences among them, and any other requirements you have. Behind the scenes is linear programming software that designs an exercise program to meet your criteria.

Suppose that a client at such a site has indicated that her primary fitness goals are relaxation and enjoyment, but that she also wants to earn at least 36 aerobic “points” a week (the points are how the site measures the benefit of the exercise in strengthening the heart and lungs). From the on-screen menus she’s indicated that her preferred exercises are jogging and cycling, which she does at rates of 6 and 18 miles per hour respectively. *An hour of jogging earns 12 aerobic points and an hour of cycling earns 9 aerobic points.* When asked to rank her preferences between these exercises she’s indicated that she prefers cycling to jogging and so wants to spend at least as much time riding as jogging. Finally she’s indicated that she is enrolled in an “Exercise your way to Vancouver” event, and to complete it she needs to cover at least 54 miles per week. To top it all off she’s a busy person and wants to meet all these goals in as few hours per week as possible.

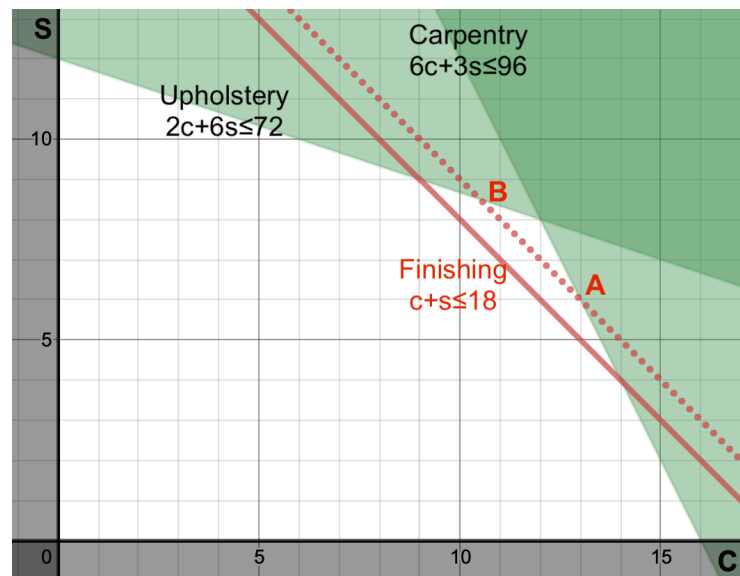
Formulate this as a linear programming problem and solve it to produce an exercise program that meets her requirements.

2. (10 marks) Shadow Prices

In addition to solving a linear programming problem, we can also determine the effect of each constraint on the final solution. One way to express that effect is through the shadow price of the constraint. The shadow price is the price you would be willing to pay to loosen that constraint one notch. For example, in our original furniture manufacturing problem the finishing constraint is $c + s \leq 18$. Remember that the number 18 represents the total number of hours available each day for finishing operations. Suppose you could buy one more hour of finishing; what is the most you should be willing to pay for it? That amount is its shadow price.

- a) If we increase the number of available finishing hours from 18 to 19, what is the new finishing constraint?
- b) What is the linear equation for this constraint?

Here is the feasible set for this problem with the original finishing constraint and the modified constraint. As you can see the modified constraint moves two of the vertices of the feasible set to new positions A and B.



- c) Find the coordinates of the points A and B.
- d) What is the new maximum profit? (Hint: Does one of these points represent a new maximum?)
- e) How much larger is the new profit than the original profit?

Your answer in e) is the shadow price for finishing shop hours. It is the most you should pay for another hour of finishing, because at this value you break even. If you can buy an hour of finishing for less than this your total profit goes up, but if it costs you more than this your profit goes down.

3. (10 marks) About those moose

Did we miss a constraint?

In modelling the foraging moose we assumed they could eat twenty-four hours a day. We never said this explicitly¹, but by not including time as a factor in our model we effectively allowed 24/7 eating. The thing is they can't. Moose can't spend too long feeding in water or they experience hypothermia. They can't spend too long feeding in the open or they overheat. And the process of rumination² requires a significant amount of time during which they cannot be feeding. Belovsky found that rather than a limit on total feeding time moose have separate daily limits on the time that can be spent feeding on aquatic plants and on terrestrial plants of 150 min/day and 256 min/day respectively. The question is: can they achieve the energy maximum within these constraints? To find out we need to add a couple more constraint lines to our graph and see if the optimum we previously found is still in the revised feasible region.

How does the time constraint work? The aquatic feeding time limit of 150 min, puts a hard limit on the amount of aquatics the moose can harvest. To calculate this limit we divide the time that can be spent feeding by the cropping rate for aquatic feeding (0.05 min/g):

$$A_{max} = \frac{150 \frac{\text{min}}{\text{day}}}{0.05 \frac{\text{min}}{\text{g}}} = 3,000 \frac{\text{g}}{\text{day}}$$

Since this is a maximum possible value it constrains A to be less or equal to this,

$$A \leq 3,000$$

a) Plot this line on the graph. Does it change the feasible region?

b) Go through the same process for the constraint on terrestrial feeding. Does it change the feasible region?

The plight of cows with calves

The data in the table we used was for the “average moose”. As Belovsky explains “The average Isle Royale moose is assumed to weigh 464 kg, is 50% bull and 50% cow and in terms of reproduction it is 54% with calf and 46% bull or barren.” A peculiar, and theoretical, animal!

¹ Implicit assumptions are the most dangerous, because often we are not even aware we have made them. This is one reason a common exercise in modelling is to take time to reflect and try to explicitly all the assumptions underlying a model. The results can be surprising. And sometimes disheartening.

² “a moose as a ruminant must spend time regurgitating its food and remasticating it for the symbiotic microorganisms in its rumen to achieve some level of digestibility.” Belovsky, Gary E. Diet optimization in a generalist herbivore: The moose. *Theoretical Population Biology* 14, 105-134 (1978).

Consider a more concrete animal: a cow with a calf. This requires changes to each of the constraints in our model.

1) *Energy requirements*. Lactation requires extra energy. The formula for the daily energy requirement for a bull or barren cow moose is $E = 2 \times 70W^{0.75}$ where W is the moose's weight. Whereas for a cow moose with calves it is $E = 2.7 \times 70W^{0.75}$.

2) *Sodium demand*. A cow with calves requires an additional 1.18 g/day/calf³ of sodium.

3) *Digestive capacity*. A cow moose has a stomach that is 1.2 times larger than that of a bull of the same weight, so the limitation of 32,900 wet g/day has to be increased.

c) What are the new constraint inequalities?

d) Plot them and identify the new feasible region.

e) How does it compare in size to the feasible region for the average moose? Calf mortality on Isle Royale is around 60%. Is this consistent with this result? Explain.

³ The strange unit g/day/calf is read "grams per day per calf"