

Exercise set 4.2: Simplex Method Skills for Non-standard Problems

Write the initial simplex tableau for each of these linear programming problems.

1. Maximize $p = 40a + 30b$ subject to the constraints $a + b \leq 5$, $-2a + 3b \geq 12$, $a \geq 0, b \geq 0$

$$\begin{aligned}
 p &= 40a + 30b \Rightarrow p - 40a - 30b = 0 \\
 a + b &\leq 5 \Rightarrow a + b + u = 5 \\
 -2a + 3b &\geq 12 \Rightarrow \text{negate } \because \geq \Rightarrow 2a - 3b \leq -12 \\
 &\Rightarrow 2a - 3b + v = -12
 \end{aligned}$$

$$\begin{array}{c} u \\ v \\ p \end{array} \left[\begin{array}{cccc|c} a & b & u & v & p \\ 1 & 1 & 1 & 0 & 5 \\ 2 & -3 & 0 & 1 & -12 \\ -40 & -30 & 0 & 0 & 0 \end{array} \right]$$

2. Minimize $C = 3m + n$ subject to the constraints $m + n \geq 3$, $2m \geq 5$, $m \geq 0, n \geq 0$

$$\begin{aligned}
 C &= 3m + n \Rightarrow C - 3m - n = 0 \Rightarrow \text{negate } \because \text{minimize} \Rightarrow -C + 3m + n = 0 \\
 m + n &\geq 3 \Rightarrow \text{negate } \because \geq \Rightarrow -m - n \leq -3 \Rightarrow -m - n + u = -3 \\
 2m &\geq 5 \Rightarrow \text{negate } \because \geq \Rightarrow -2m \leq -5 \Rightarrow -2m + v = -5
 \end{aligned}$$

$$\begin{array}{c} u \\ v \\ C \end{array} \left[\begin{array}{cc|cc|c} m & n & u & v & C \\ -1 & -1 & 1 & 0 & -3 \\ -2 & 0 & 0 & 1 & -5 \\ 3 & 1 & 0 & 0 & 0 \end{array} \right]$$

3. Minimize $f = 2x + 7y$ subject to the constraints $2x + 5y \geq 30$, $-3x + 5y \geq 5$, $8x + 3y \leq 101$, $-9x + 7y \leq 42$, $x \geq 0, y \geq 0$

$$\begin{aligned}
 f &= 2x + 7y \Rightarrow f - 2x - 7y = 0 \Rightarrow \text{negate } \because \text{min} \Rightarrow -f + 2x + 7y = 0 \\
 2x + 5y &\geq 30 \Rightarrow \text{negate } \because \geq \Rightarrow -2x - 5y \leq -30 \Rightarrow -2x - 5y + u = -30 \\
 -3x + 5y &\geq 5 \Rightarrow \text{negate } \Rightarrow 3x - 5y \leq -5 \Rightarrow 3x - 5y + v = -5 \\
 8x + 3y &\leq 101 \Rightarrow 8x + 3y + w = 101 \\
 -9x + 7y &\leq 42 \Rightarrow -9x + 7y + z = 42
 \end{aligned}$$

} Already $\leq$ constraints so no need to negate.

$$\begin{array}{c} u \\ v \\ w \\ z \end{array} \left[\begin{array}{cc|cc|cc|c} x & y & u & v & w & z & f \\ -2 & -5 & 1 & 0 & 0 & 0 & -30 \\ 3 & -5 & 0 & 1 & 0 & 0 & -5 \\ 8 & 3 & 0 & 0 & 1 & 0 & 101 \\ -9 & 7 & 0 & 0 & 0 & 1 & 42 \end{array} \right]$$

$$f \begin{bmatrix} 2 & 7 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

Circle the pivot column and pivot element for each of these initial simplex tableaux.

4.
$$\begin{array}{c|ccccccc} & x & y & z & u & v & w & f \\ \hline u & 1 & -1 & 1 & 1 & 0 & 0 & -3 \\ v & 2 & 2 & 1 & 0 & 1 & 0 & 4 \\ w & 1 & -1 & 0 & 0 & 1 & 0 & 1 \\ f & 4 & 2 & 1 & 0 & 0 & -1 & 0 \end{array}$$

① Start → (Row 1, Column 1)
 ② Choose column (Column 1)
 ③ Find Ratios:
 $-3/-1 = 3$
 $4/2 = 2$ ← Smallest non-negative
 $1/-1 = -1$

④ Pivot here (Row 2, Column 1)

5.
$$\begin{array}{c|ccccccc} & x & y & u & v & w & f \\ \hline u & -2 & 1 & 1 & 0 & 0 & 0 & -16 \\ v & 1 & 2 & 0 & 1 & 0 & 0 & 11 \\ w & -3 & 3 & 0 & 0 & 1 & 0 & -15 \\ f & 30 & 50 & 0 & 0 & 0 & 1 & 8 \end{array}$$

① Start → (Row 1, Column 1) or (Row 3, Column 1)
 ② Column (Column 1)
 ③ Ratios:
 $-16/-2 = 8$
 $11/1 = 11$
 $-15/-3 = 5$ ← Smallest

④ Pivot here (Row 3, Column 1)

6.
$$\begin{array}{c|ccccccc} & x & y & z & u & v & f \\ \hline u & 5 & 2 & -2 & 1 & 0 & 0 & -6 \\ v & 1 & 2 & -3 & 0 & 1 & 0 & -12 \\ f & 10 & 12 & 12 & 0 & 0 & 1 & 0 \end{array}$$

Start → (Row 1, Column 3) or (Row 2, Column 3)
 ② Column (Column 3)
 ③ Ratios:
 $-6/-2 = 3$ ← Smallest
 $-12/-3 = 4$

④ Pivot here (Row 1, Column 3)