

Exercise set 5.3: Log and exponential models

1. Solve the following equations for x :

a. $3^x = 5$ $\log 3^x = \log 5 \Rightarrow x \log 3 = \log 5 \Rightarrow x = \frac{\log 5}{\log 3} \approx 1.46497\dots$

b. $6^{3x+1} = 30$ $(3x+1) \log 6 = \log 30 \Rightarrow 3x+1 = \frac{\log 30}{\log 6} \Rightarrow 3x = \frac{\log 30}{\log 6} - 1 \Rightarrow x = \frac{\frac{\log 30}{\log 6} - 1}{3} \approx 0.3$

c. $4(1.5^{2x-1}) = 8$ $(2x-1) \log 1.5 = \log(8/4) \Rightarrow x = \left(\frac{\log 2}{\log 1.5} + 1 \right) / 2 \approx 1.355$

2. Solve the following equations for x :

a. $\log x = 6.12$ $x = 10^{6.12} = 1,318,256.739$

b. $\log(2x+3) = 2$ $2x+3 = 10^2 \Rightarrow 2x = 10^2 - 3 \Rightarrow x = \frac{10^2 - 3}{2} = 48.5$

c. $\log_4(x+1) = 6$ $x+1 = 4^6 \Rightarrow x = 4^6 - 1 = 4,095$

3.

What is the equation of the exponential function that passes through the points (1,6) and (3,24)?

$24 = A \cdot b^3$

$y = A b^x$

① $6 = A \cdot b^1$

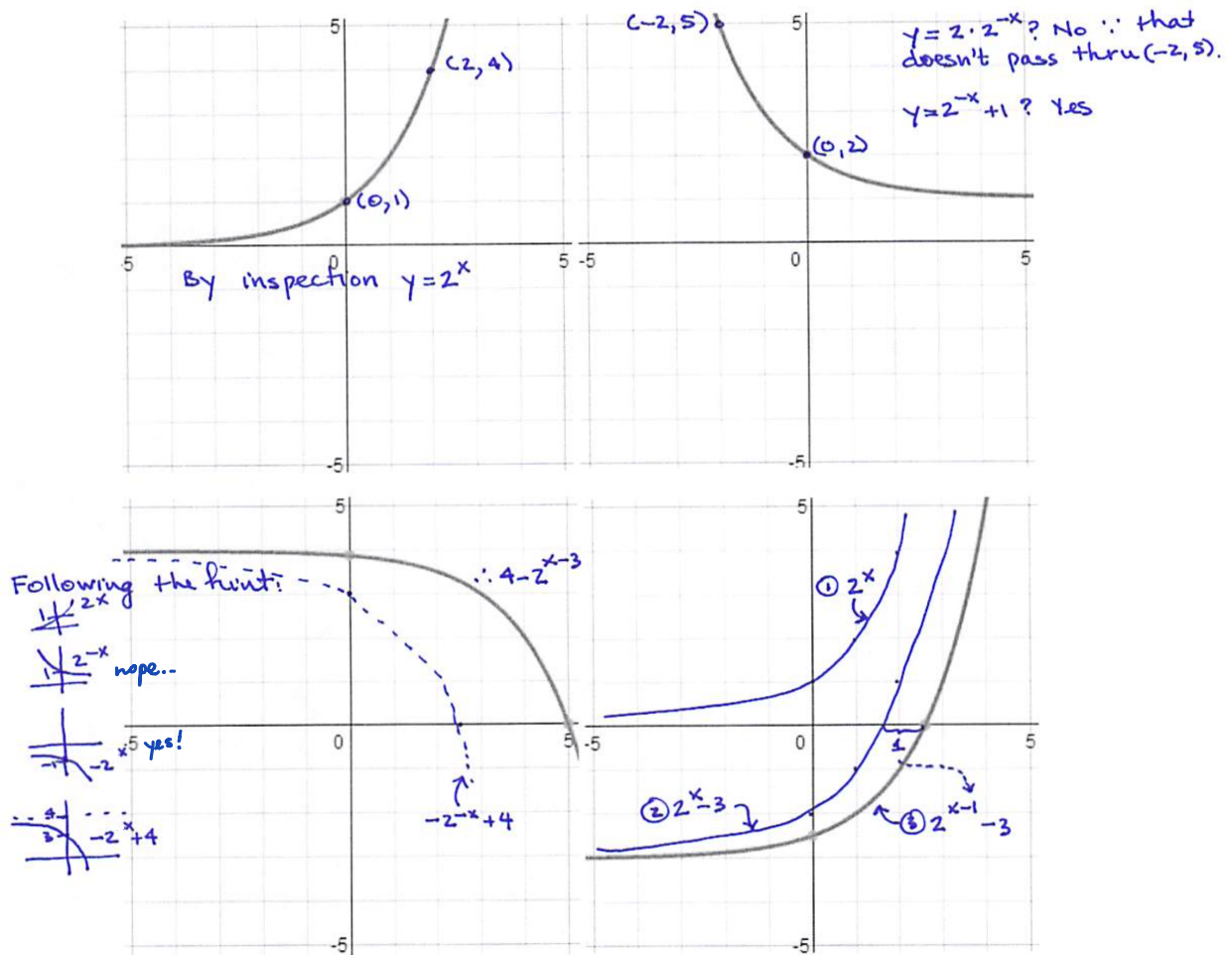
① $\Rightarrow A = 6/b$

Sub ① in ②: $24 = \frac{6}{b} \cdot b^{3^2} \Rightarrow b^2 = 4 \Rightarrow b = 2$

Sub back in ①: $A = 6/2 = 3$

$\therefore y = 3 \cdot 2^x$

4. What are the equations of the functions below? (Hint: They are all based on 2^x .)



5. In the early stages of the AIDS epidemic during the 1980s, the number of cases in the US was increasing by about 50% every six months. By the start of 1983 there were approximately 1600 AIDS cases in the U.S. $y(6) = 1.5 y_0 = y_0 b^6 \Rightarrow b = \sqrt[6]{1.5} \approx 1.069913194$
 $y_0 = 1600$

a. How many people would you have predicted would be infected by October 1, 1986? $y(45)$

$$y(45) \approx 1600 (1.069913194)^{45} \approx 33,481$$

b. By December 31, 1986? (The actual number of cases on December 31, 1986 was around 41,700.) $y(48)$

$$y(48) \approx 1600 \cdot 1.069913194^{48} \approx 41,006$$

6. Soon after taking an aspirin tablet a patient has absorbed 300 mg of the drug. If half of the aspirin in the bloodstream is removed every two hours, how much aspirin will be in the bloodstream after 5 hours? When will the amount of aspirin in the bloodstream be 100g? $y_0 = 300$

$$y(2) = \frac{1}{2} y_0$$

$$y_2 = 0.5 y_0 = y_0 b^2 \Rightarrow b = \sqrt{0.5}$$

$$y(5) = 300 (\sqrt{0.5})^5 \approx 53.033$$

$$t \text{ for } y = 100?$$

$$100 = 300 (\sqrt{0.5})^t$$

$$t = \frac{\log(100/300)}{\log \sqrt{0.5}} \approx 3.1699 \text{ hrs}$$

7. The amount of radium-226 remaining in a sample that originally contained A grams is approximately $C(t) = A(0.999567)^t$ where t is time in years. What is the half-life of radium-226? i.e. t for $C = A/2$

$$\frac{A}{2} = A(0.999567)^t$$

$$t = \frac{\log \frac{1}{2}}{\log 0.999567} \approx 1600 \text{ yrs}$$

8. The half-life of strontium-90 is 28 years. How long will it take 80% of a sample of strontium-90 to decay? $T = 28$

$$T = 28 = \frac{-\log 2}{\log b}$$

$$\log b = \frac{-\log 2}{28}$$

$$b = 10^{\frac{\log 2}{28}} \approx 1.025064212$$

$$t \text{ for } y = 0.2 y_0?$$

$$0.2 y_0 = y_0 (1.025064212)^t$$

$$t = \frac{\log 0.2}{\log 1.025064212} \approx 65 \text{ years}$$

9. and 10.

Sketch the function $f(x) = \frac{9}{1+3.5(2^{-x})}$

$\nwarrow N$
 $\nearrow A \quad \nwarrow b$

Sketch the function $f(x) = \frac{6}{1+3(0.5^{-x})}$

$y_{\infty} - \text{int} = \frac{6}{1+3} = 1.5$

$b < 1 \quad \therefore$ shrinking not growing

