

The Allee effect

Exponential and logistic growth models assume even tiny populations can grow and prosper, but this is not always the case in the real world. Some populations decline to extinction unless they stay above a critical value.

The American ecologist Warder Clyde Allee proposed a model to describe this in terms of growth rate. In one specific case the growth rate was found to be,

$$f(N) = -N^3 + 11N^2 - 24N$$

for values of population density N from 0 to 9 (inclusive).



What are the maximum and minimum growth rates and at what population sizes do they occur?

① stationary

$$f'(N) = -3N^2 + 11 \cdot 2N - 24 = -3N^2 + 22N - 24 \stackrel{!}{=} 0$$

$$N = \frac{-22 \pm \sqrt{22^2 - 4(-3)(-24)}}{2(-3)}$$

$$= \frac{-22 \pm \sqrt{22^2 - 288}}{-6}$$

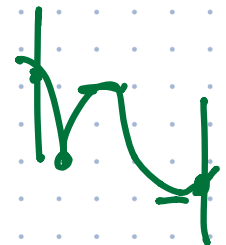
$$= \frac{-22 \pm 14}{-6}$$

$$\frac{-8}{-6} = \frac{4}{3}$$

N

$$\frac{-36}{-6} = +6$$

N



② $f'(N)$ DNE? None.

$$f\left(\frac{4}{3}\right) = -\left(\frac{4}{3}\right)^3 + 11\left(\frac{4}{3}\right)^2 - 24\left(\frac{4}{3}\right) = -14.8 \dots$$

$$f(6) = -6^3 + 11 \cdot 6^2 - 24(6) = +36$$

③ Endpoints.
 $f(0) = 0$

$$f(9) = -54$$

