

How large a web to weave?



Some species of spiders weave webs to capture prey which they then ingest. To ensure capturing prey they would like to weave a large web. On the other hand it takes energy and nutrients both to produce the silk, and to construct the web from it. A lazy spider might not build a web large enough to capture sufficient prey to meet its energetic requirements. A careless spider might expend more nutrients and energy building a vast web than it can recoup by eating its captured prey. Somewhere in between these extremes is a sweet spot where the difference between the energy spent, and the energy received is a maximum. Suppose that the energy in is roughly proportional to the area of the web. The energy out though is proportional to the cube of the radius of the web (because a larger web requires not just longer, but also thicker strands of silk).

How large a web should our spider build?

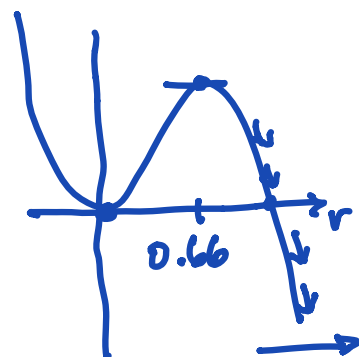
Maximize $E = E_{in} - E_{out}$

$$\begin{aligned} E_{in} &\propto A \\ &\propto \pi r^2 \\ &= k_1 \pi r^2 \end{aligned}$$

$$\begin{aligned} E_{out} &\propto r^3 \\ &= k_2 r^3 \end{aligned}$$

$$E = k_1 \pi r^2 - k_2 r^3$$

← Formulation



check ① stationary points
② end points $r=0$,
③ other critical points

$$\textcircled{1} \frac{dE}{dr} = k_1 \pi \cdot 2r - k_2 \cdot 3r^2 \stackrel{!}{=} 0$$

$$r(2k_1 \pi - 3k_2 r) \stackrel{!}{=} 0$$

$r=0$ / $2k_1 \pi - 3k_2 r = 0$

$$r = \frac{+2k_1 \pi}{+3k_2}$$

$$k_1 = 0.6$$

$$k_2 = 1.9$$

$$\approx \underline{\underline{0.66 \text{ m}}}$$

③ $\wedge \rightarrow \searrow \}$

② $r=0, r \rightarrow \infty$ Not optimums