Exercise set 7.1: Applications of the derivative

1. **Photosynthesis** The rate (in mg of carbon/m^3/h) at which photosynthesis takes place for a species of phytoplankton is modeled by the function

$$P = \frac{100I}{I^2 + I + 4}$$

where I is the light intensity (measured in thousands of foot-candles). For what light intensity is P a maximum?

2. **Crop yield** A model used for the yield Y of an agricultural crops as a function of the nitrogen level N in the soil is

$$Y = \frac{27N}{1 + N^2}$$

What nitrogen level gives the best yield?

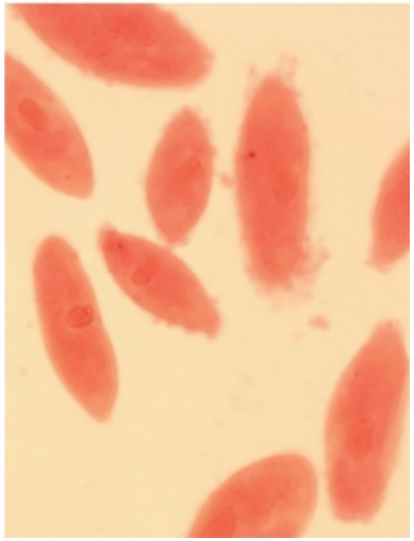
3. **Disease virulence** The Kermack-McKendrick model for infectious disease transmission can be used to predict the population size P as a function of the disease's virulence. Virulence measures the deadliness of a disease. The population size P is high when virulence v is low because it is not very deadly. Surprisingly P is also high when v is high because then the disease is so deadly it kills people before they can pass it on, so fewer people actually get infected. Suppose,

$$P(v) = \frac{10 + v + v^2}{1 + v}$$

for values of v from 0 to 10 inclusive. What virulence values produce the largest and smallest population sizes? What are those populations?

4. **Gause's logistic model** In an experiment with the protozoan *Paramecium* the biologist G.F. Gause modeled the protozoan population size with the logistic function

$$P(t) = \frac{61}{1 + 31e^{-0.7944t}}$$

Where t is measured in days. According to this model, how fast was the population growing after 8 days?

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