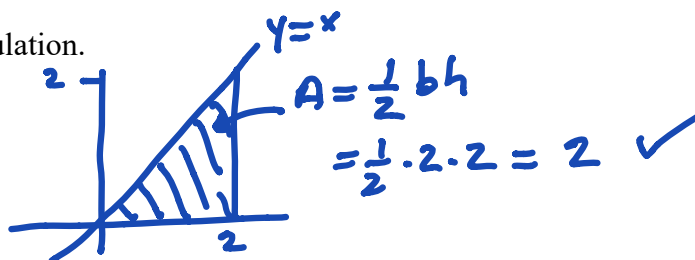


Exercise set 10.2: Evaluating Integrals

Evaluate each of these integrals.

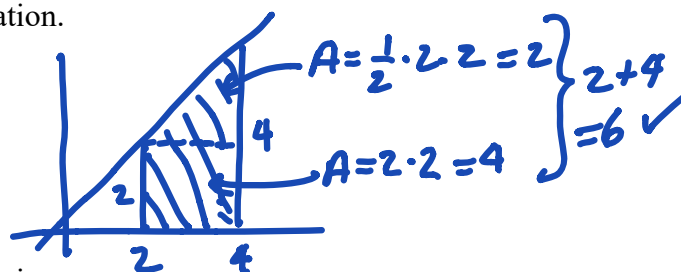
1. $\int_0^2 x \, dx$ Verify your answer using a geometric calculation.

$$= \left[\frac{x^2}{2} \right]_0^2 = \frac{2^2}{2} - \frac{0^2}{2} = 2$$



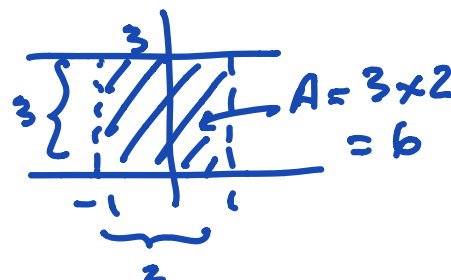
2. $\int_2^4 x \, dx$ Verify your answer using a geometric calculation.

$$= \left[\frac{x^2}{2} \right]_2^4 = \frac{4^2}{2} - \frac{2^2}{2} = 8 - 2 = 6$$



3. $\int_{-1}^1 3 \, dx$ Verify your answer using a geometric calculation.

$$= 3x \Big|_{-1}^1 = 3 \cdot 1 - 3(-1) = 3 + 3 = 6$$



4. $\int_{-1}^1 x^2 \, dx$

$$= \left[\frac{x^3}{3} \right]_{-1}^1 = \frac{1^3}{3} - \frac{(-1)^3}{3} = \frac{1}{3} - \frac{-1}{3} = \frac{2}{3}$$

5. $\int_0^1 (x^3 + 2) \, dx$

$$= \left[\frac{x^4}{4} + 2x \right]_0^1 = \left(\frac{1^4}{4} + 2 \cdot 1 \right) - \left(\frac{0^4}{4} + 2 \cdot 0 \right) = \frac{1}{4} + 2 = 9/4$$

6. $\int_1^2 (5x^3 - x) \, dx$

$$\begin{aligned} &= \left[\frac{5x^4}{4} - \frac{x^2}{2} \right]_1^2 = \left(\frac{5 \cdot 2^4}{4} - \frac{2^2}{2} \right) - \left(\frac{5 \cdot 1^4}{4} - \frac{1^2}{2} \right) \\ &= (20 - 2) - \left(\frac{5}{4} - \frac{1}{2} \right) \\ &= 18 - 3/4 = 17 \frac{1}{4} \end{aligned}$$

$$7. \int_0^4 \sqrt{x} dx = \int_0^4 x^{1/2} dx$$

$$= \left[\frac{2}{3} x^{3/2} \right]_0^4 = \frac{2}{3} 4^{3/2} - \frac{2}{3} 0^{3/2} = \frac{2}{3} \cdot 8 = \frac{16}{3}$$

$$8. \int_1^3 (2x-1)^3 dx$$

$$= \left[\frac{1}{2} \cdot \frac{1}{4} (2x-1)^4 \right]_1^3 = \frac{1}{8} (2 \cdot 3 - 1)^4 - \frac{1}{8} (2 \cdot 1 - 1)^4 = \frac{1}{8} [5^4 - 1] = 78$$

$$9. \int_2^{3.5} (2.9x^4 - 4.1x^2 + 3) dx$$

$$= \left[2.9 \frac{x^5}{5} - 4.1 \frac{x^3}{3} + 3x \right]_2^{3.5} \approx 242.9$$

$$10. \int_{0.5}^{0.8} (4.2x^3 - 5.9x^2 + 2.3x - 7.1) dx$$

$$= \left[4.2 \frac{x^4}{4} - 5.9 \frac{x^3}{3} + 2.3 \frac{x^2}{2} - 7.1x \right]_{0.5}^{0.8} \approx \underline{-3.44} \\ -1.078145$$

$$11. \int_0^1 e^x dx$$

$$= e^x \Big|_0^1 = e^1 - e^0 = e - 1 \approx 1.718282$$

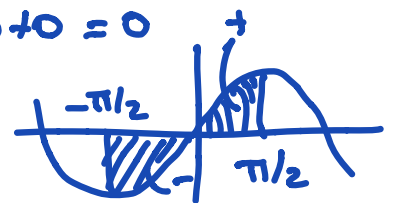
$$12. \int_5^{10} (e^{2x} - x) dx$$

$$= \left[\frac{1}{2} e^{2x} - \frac{x^2}{2} \right]_5^{10} = \left(\frac{e^{20}}{2} - \frac{10^2}{2} \right) - \left(\frac{e^{10}}{2} - \frac{5^2}{2} \right) = \frac{e^{20} - e^{10} - 100 + 25}{2}$$

$$13. \int_{-\pi/2}^{\pi/2} \sin x dx \text{ Draw a picture to verify your result.}$$

$$= -\cos x \Big|_{-\pi/2}^{\pi/2} = -\cos \frac{\pi}{2} - -\cos \left(-\frac{\pi}{2} \right) = 0 + 0 = 0$$

Areas equal
and opposite so
cancel.



$$14. \int_0^{\pi} (x - e^x + \cos x) dx$$

$$= \left[\frac{x^2}{2} - e^x + \sin x \right]_0^{\pi} = \left(\frac{\pi^2}{2} - e^{\pi} + \sin \pi \right) - \left(\frac{0^2}{2} - e^0 + \sin 0 \right)$$

$$= \frac{\pi^2}{2} - e^{\pi} + 1 \approx -12.27$$

$$-17.2058904$$

$$15. \int_{\pi/2}^{\pi} \left(\sin x + \frac{4}{x^3} \right) dx$$

$$= \left[-\cos x - \frac{2}{x^2} \right]_{\pi/2}^{\pi} = \left(-\cos \pi - \frac{2}{\pi^2} \right) - \left(-\cos \frac{\pi}{2} - \frac{2}{(\pi/2)^2} \right)$$

$$= 1 - \frac{2}{\pi^2} + \frac{8}{\pi^2}$$

$$= 1 + \frac{6}{\pi^2}$$

$$\approx 1.608$$

$\therefore \frac{4}{x^3} = 4x^{-3}$ power rule $\Rightarrow \int = ? x^{-3+1} = 2x^{-2}$