

1. (5 marks) *Smelts*. A recent survey of mitochondrial DNA variation in smelts in a lake revealed that two haplotypes (genotypes) were present. Forty percent of individual fish in the population were haplotype A. The remaining 60 percent were haplotype B. If an experimenter were to sample 10 fish from this population, what is the probability that none are haplotype A? At least 5 are haplotype B?
2. (10 marks) *Paradise flycatchers*. A population of paradise flycatchers has 80% brown males and 20% white. You capture five male flycatchers at random.
 - a. What is the chance that at least two are white?
 - b. You need to band at least four white ones. How many should you plan to capture to have a 50% chance of getting four white ones?
 - c. Your supervisor wants you to be “almost certain” of banding four white ones. How many should you plan to capture to have a 90% chance of getting four white ones?
3. (10 marks) *About that probability*. Estimate the probability that in the mark-recapture problem done in class the value of N is between:
 - a. 55 and 65 inclusive¹.
 - b. 50 and 70 inclusive.
4. (10 marks) *Pitcher problems and linear equations*

There is more than one way to solve most problems and pitcher problems are no exception. For example the problem of measuring volumes of liquid using a 3-liter and a 7-liter pitcher can be formulated as an algebraic problem, specifically by the expression

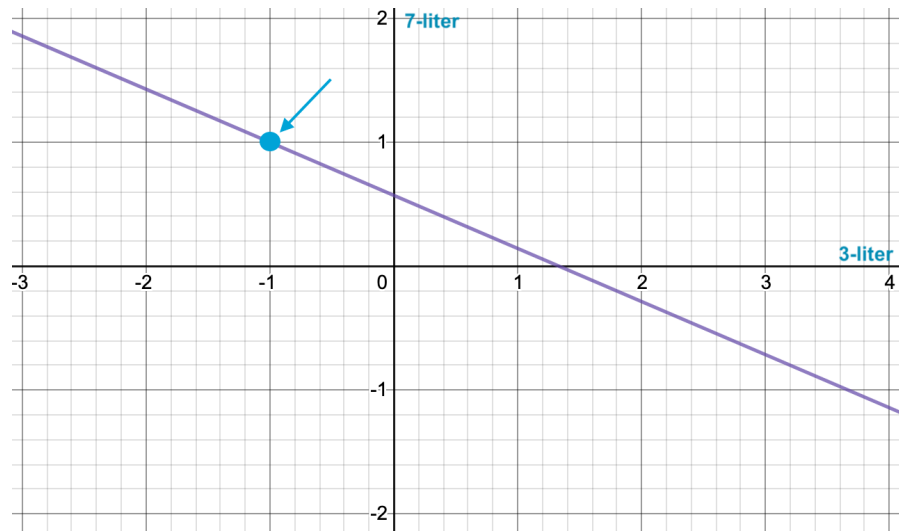
$$3x + 7y$$

where x represents the number of times the 3-liter pitcher is filled, and y represents the number of times we fill the 7-liter pitcher. A positive value means the pitcher is filled while a negative value means it is emptied. For example $x = +1$ and $y = +1$ corresponds to filling both pitchers, and making 10 gallons. Here's how to use the expression. Let's say we want to measure 4 liters. We plot the equation,

$$3x + 7y = 4$$

¹ *inclusive* means including the endpoints, i.e. including 55 and 65

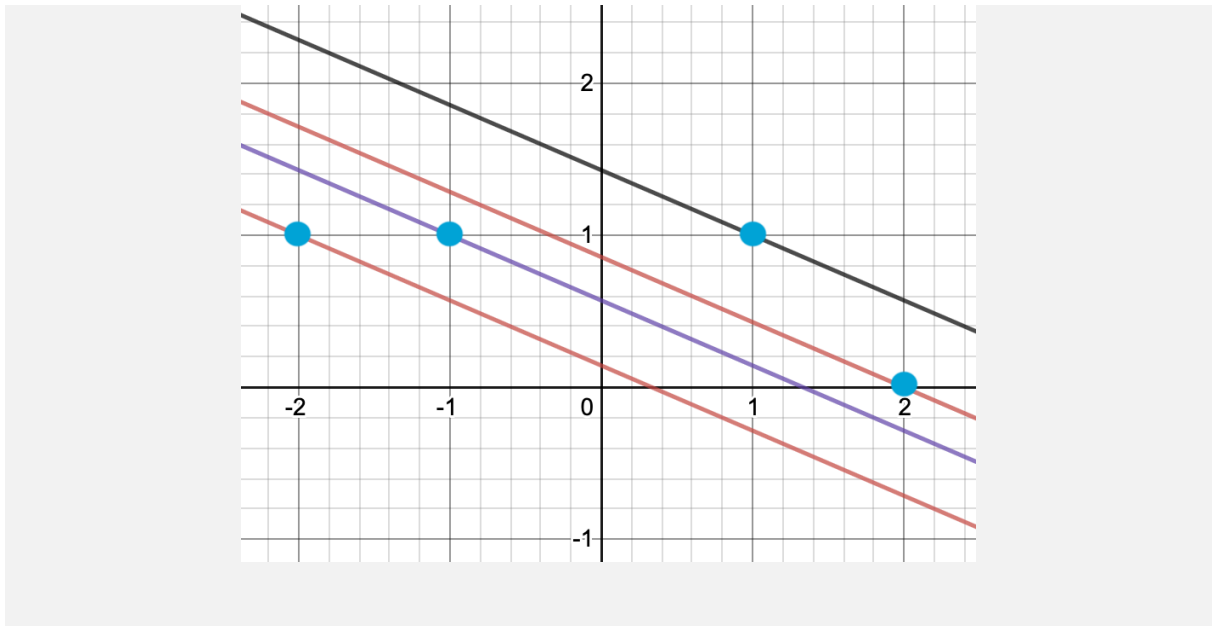
And look for places the line crosses the intersections of integer coordinates. There is only one in this case at $(-1, 1)$.



What does this mean? Well $x = -1$ and $y = +1$ so it means we should empty the 3-liter pitcher once (-1) and fill the 7-liter pitcher once $(+1)$. What actions does that imply? After all we can't start by emptying the 3-liter pitcher because there's nothing in it to start with. It means we should fill the 7-liter pitcher, pour from it into the 3-liter pitcher, and then empty the 3-liter pitcher.

- By setting the expression $3x + 7y$ to every value from 1 through 10 and plotting the equations that result, verify which amounts can be measured with these two pitchers, and write out the filling and pouring instructions for each amount.

Here are some more examples of lines with their integer intersections marked by circles. (Why only integer intersections? Because we can't partly fill or empty the pitchers.)



5. (5 marks) *Seemingly simple*

- a) Draw a graph with 4 vertices of degree 2.
- b) Draw a graph with three vertices of degree 3, and two vertices of degree 2.