

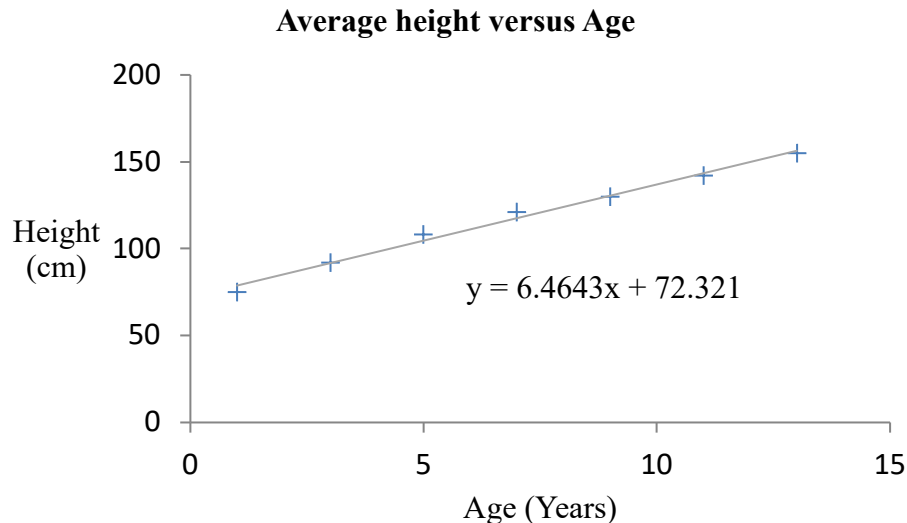
Name: _____

ALES 291
Final Examination
April 2014

Instructions:

- ◆ You have three (3) hours to complete the examination.
- ◆ The value of all questions is shown — choose well, and use your time wisely!
- ◆ A box **must** be placed around your final answer to each question.
- ◆ Part marks can only be given if your rough work is shown. No marks will be given for unsupported answers.
- ◆ Some questions specify the method to use to find the solution. In these cases you **must** use the specified method to receive any marks for the question. (You may however check your answer by using a second method).
- ◆ If you are unsure of anything, **ask!**

1. (10 marks) Organism growth is not generally linear. Instead organisms often grow in “spurts” that depend on their current environmental conditions, and on their genetics. Although few growth processes are linear for their entire cycle, many have approximately-linear portions. Here for example is some data and a linear model for the growth of juvenile humans¹.



- a) How much does the typical child grow each year?
- b) Based on the model what is the average height of an eight year old?
- c) What height does the model predict for a newborn baby?

¹ Data source: David N. Holvey, editor, *The Merck Manual of Diagnosis and Therapy* (1987) 15th ed., Merck Sharp & Dohme Research Laboratories, Rahway, NJ.

2. (20 marks) A small toy company makes two kinds of stuffed animals. A stuffed puppy costs \$2 in materials and requires 4 hours of labour. A teddy bear costs \$2.50 in materials and requires 3 hours of labour. The company has \$40 available for materials and already has orders for 5 animals of each type. A total of 64 labour hours are available. Find the number of stuffed animals of each type the company should make to meet the orders and maximize profit if the profit on each stuffed puppy is \$15 and on each bear is \$10.
3. (10 marks) For the Simplex tableau below, determine the next pivot element and pivot the tableau about it.

	x	y	z	u	v	w	p	
u	1	1	1	1	0	0	0	3
v	2	2	1	0	1	0	0	4
w	1	-1	0	0	0	1	0	0
p	-4	-2	1	0	0	0	1	0

	x	y	z	u	v	w	p	

4. (10 marks) The Cutting Edge Knife Company manufactures paring knives and jackknives. The table below summarizes their production situation.

	Paring knife	Jackknife	Available resources
Labour hours	3 hours	6 hours	90 hours
Steel	7 units	5 units	138 units
Wood	4 units	3 units	120 units
Profit	\$9	\$5	

And here is a Simplex tableau for this system:

	x	y	u	v	w	p	
y	0	1	$7/27$	$-1/9$	0	0	8
x	1	0	$-5/27$	$2/9$	0	0	14
w	0	0	$-1/27$	$-5/9$	1	0	40
p	0	0	$20/27$	$1/9$	0	1	82

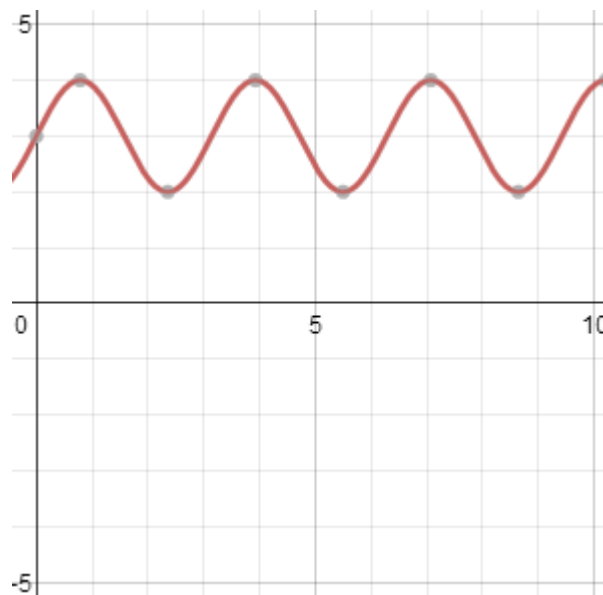
Where x = paring knives, y = jackknives, u = labour hour slack, v = unused steel, w = unused wood, and p represents profit.

- a) How many paring knives and jackknives are produced in the production plan the tableau represents?
- b) Does this tableau represent a maximum profit production plan? Why?
- c) What are the shadow prices for labour hours? One unit of steel? One unit of wood?

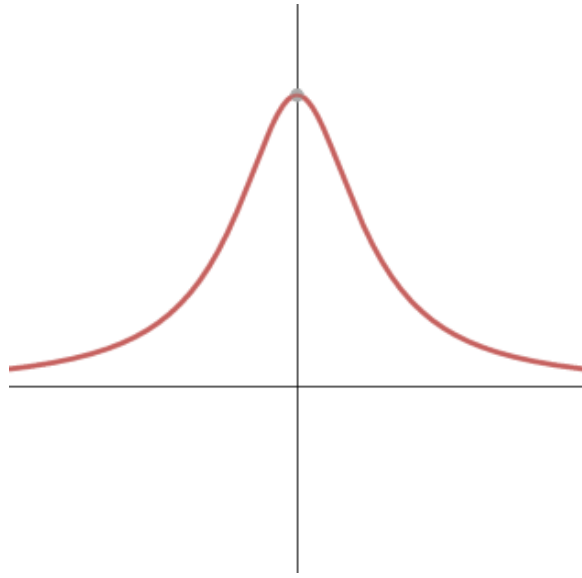
5. (10 marks) Sketch the functions:

- a) $y = 2(x+1)^2$
- b) $y = 2 - e^x$

6. (5 marks) What is the equation of the function below?



7. (5 marks) Sketch the derivative of each of the function.



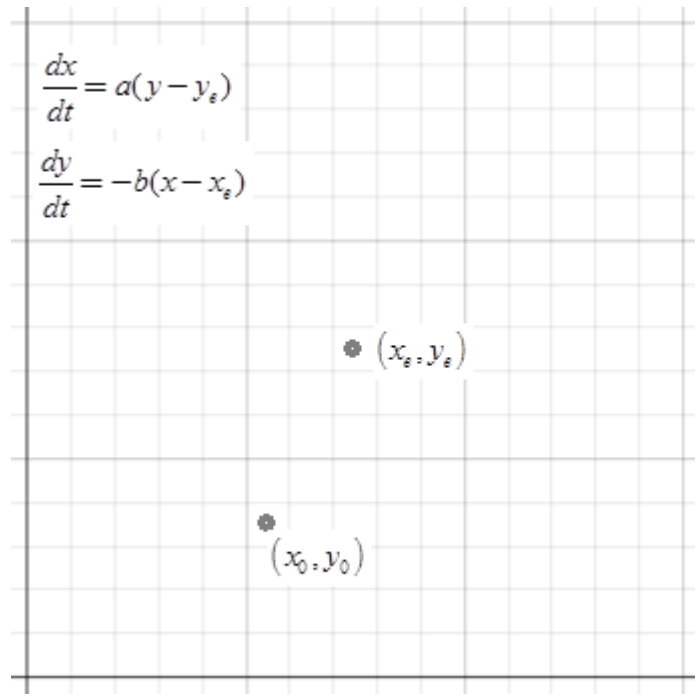
8. (10 marks) Find the derivative of:

a. $f(x) = 3x^2 - 2x + 5$

b. $y = e^x - \sin x$

9. (5 marks) Is $y = \sin t + \cos t$ a solution to the differential equation $y'' + y = 0$?

10. (5 marks) The graph below shows an equilibrium position for the system, (x_e, y_e) , an initial position, (x_0, y_0) , and the derivatives of the x and y quantities. Sketch the system trajectory that results.



11. (10 marks) **Personal Income and Expenditures.** Suppose someone's annual income is represented by $I(t) = 50,000(1.03)^t$ while their costs can be modeled by the function $C(t) = 45,000 + 50t^2$. Find the area between these functions for $0 \leq t \leq 35$? What does your answer represent?
12. (10 marks) A card is to be dealt from a shuffled deck.
- What is the probability that it will be the Ace of Clubs?
 - What is the probability that it will be a Club?
 - What is the probability that it will be a face card?
 - What is the probability that it will be a face card and a club?
 - What is the probability that it will be a face card or a club?
13. (15 marks) Suppose that in 2005 it was estimated that 10% of Olympic athletes use steroids. The test currently being used then to detect steroids was 93% effective in correctly detecting steroids in users. It yielded false positives in only 2% of the tests. If a country's best weightlifter tested positive, what is the probability that he actually took steroids?

14. (10 marks) A coin is tossed six times.

- How many different patterns of heads and tails are possible?
- In how many patterns are there exactly 3 heads?
- In how many patterns are there at least 3 heads?

15. (5 marks) Consider the graph whose weighted adjacency matrix is:

$$\begin{array}{c}
 a \quad b \quad c \quad d \quad e \\
 \begin{array}{l}
 a \left[\begin{array}{ccccc}
 - & 4 & - & 8 & 2 \\
 b \left[\begin{array}{ccccc}
 4 & - & 3 & 7 & 1 \\
 c \left[\begin{array}{ccccc}
 - & 3 & - & - & 9 \\
 d \left[\begin{array}{ccccc}
 8 & 7 & - & 3 & 5 \\
 e \left[\begin{array}{ccccc}
 2 & 1 & 9 & 5 & -
 \end{array} \right.
 \end{array} \right.
 \end{array}
 \end{array}
 \end{array}$$

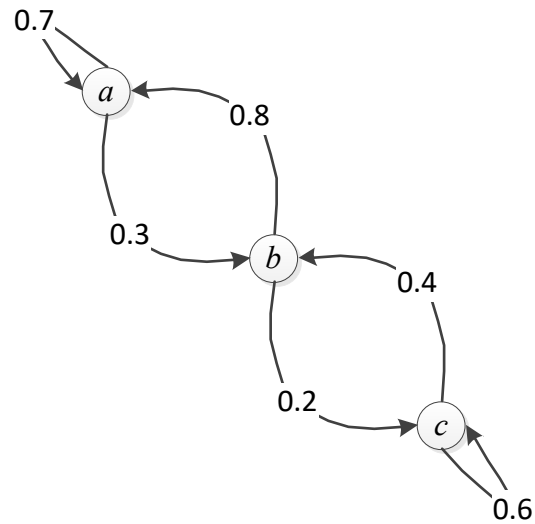
- Is it directed or undirected?
- How many vertices does it have?
- How many edges does it have?
- Are there any loops?

16. (10 marks) For the graph whose adjacency matrix is:

	A	B	C	D	E	F	G	H
A		6	12	8	4	2	6	2
B	6		2	9	4	9	5	4
C	3	5		3	11	8	6	5
D	9	7	5		9	1	3	7
E	11	6	11	9		6	3	4
F	8	9	5	1	6		4	9
G	4	5	6	3	3	1		8
H	7	4	11	7	4	9	8	

- What is the shortest route from A to C?
- What is the length of this route?

17. (20 marks) Consider the directed graph below where the vertices represent states and the weights on the edges represent the probabilities of moving from one state to another.



- Draw the state transition matrix for the graph.
 - What is the long-term stable distribution of this system?
18. (10 marks) Why is the difference of two odd numbers always even, e.g. $9 - 5 = 4$?