

What is this?

$$\begin{array}{l}
 \begin{array}{cc} \xrightarrow{c} & \xleftarrow{d} \end{array} \\
 \begin{bmatrix} 1 & 1 & 10 \\ 5 & 6 & 56 \end{bmatrix} \leftarrow \begin{array}{l} 1c + 1d = 10 \\ 5c + 6d = 56 \end{array} \\
 \begin{array}{l} \text{①} \\ \text{②} \end{array} \\
 \begin{array}{l} c + d = 10 \quad \text{①} \\ 5c + 6d = 56 \quad \text{②} \\ \hline \text{①} \times 5 - (5c + 6d = 56) \\ \hline 0c + 1d = 6 \\ \text{③} \end{array} \\
 \rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 6 \end{bmatrix} \begin{array}{l} R_2 - R_1 \times 5 \\ \dots \end{array} \\
 \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 6 \end{bmatrix} \begin{array}{l} R_1 - R_2 \\ \dots \end{array} \\
 \begin{array}{l} 1c + 0d = 4 \rightarrow c = 4 \\ 0c + 1d = 6 \rightarrow d = 6 \end{array}
 \end{array}$$

What transformations are allowed?

$$\begin{array}{l}
 x - 2y = 1 \quad \times 3 \rightarrow 3x - 6y = 3 \\
 1 \quad -2 \quad 1 \quad \times 3 \rightarrow 3 \quad -6 \quad 3
 \end{array}$$

- ① Multiply a row by a number.
- ② Add two rows together
↳ or subtract
- ③ Add or subtract a multiple of one row to another.
- ④ Interchange rows.
- ⑤ Interchange columns (take labels with them).