

3x3 Example 1

$$\begin{aligned} 1x - 3y - 2z &= 6 \\ 2x - 4y - 3z &= 8 \\ -3x + 6y + 8z &= -5 \end{aligned}$$

$$\begin{aligned} &\left[\begin{array}{ccc|c} 1 & -3 & -2 & 6 \\ 2 & -4 & -3 & 8 \\ -3 & 6 & 8 & -5 \end{array} \right] \begin{array}{l} R_2 - 2R_1 \\ R_3 + 3R_1 \end{array} \rightarrow \left(\begin{array}{ccc|c} 2 & -4 & -3 & 8 \\ 2 & -6 & -4 & 12 \\ 0 & 2 & 1 & -4 \end{array} \right) \end{aligned}$$

$$\begin{aligned} &\left[\begin{array}{ccc|c} 1 & -3 & -2 & 6 \\ 0 & 2 & 1 & -4 \\ 0 & -3 & 2 & 13 \end{array} \right] \begin{array}{l} R_1 + 3R_2 \\ R_2 \div 2 \\ R_3 + 3R_2 \end{array} \rightarrow \left(\begin{array}{ccc|c} -3 & 6 & 8 & -5 \\ 3 & -9 & -6 & 18 \\ 0 & -3 & 2 & 13 \end{array} \right) \end{aligned}$$

$$\begin{aligned} &\left[\begin{array}{ccc|c} 1 & 0 & -11/2 & 0 \\ 0 & 1 & 1/2 & -2 \\ 0 & 0 & 7/2 & 7 \end{array} \right] \begin{array}{l} R_1 + 11/2 R_3 \\ R_2 - 1/2 R_3 \\ R_3 \times 2/7 \end{array} \rightarrow \left(\begin{array}{ccc|c} 1 & -3 & -2 & 6 \\ 0 & 3 & 3/2 & -6 \\ 1 & 0 & -1/2 & 0 \end{array} \right) \end{aligned}$$

$$\begin{aligned} &\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array} \right] \begin{array}{l} R_1 + 3R_2 \\ R_2 - 2R_3 \end{array} \rightarrow \left(\begin{array}{ccc|c} 0 & -3 & 2 & 13 \\ 0 & 3 & 3/2 & -6 \\ 0 & 0 & 7/2 & 7 \end{array} \right) \end{aligned}$$

Interpretation:

$$\begin{aligned} 1x + 0y + 0z &= 1 \\ x &= 1 \\ 0x + 1y + 0z &= -3 \\ y &= -3 \\ 0x + 0y + 1z &= 2 \\ z &= 2 \end{aligned}$$

$$\frac{7}{2} \cdot \frac{2}{7} = \frac{14}{14} = 1$$

$$\frac{7}{2} \times \frac{2}{7} = 2$$

$$\rightarrow \left(\begin{array}{ccc|c} 0 & 1 & 1/2 & -2 \\ 0 & 0 & -1/2 & -1 \\ 0 & 1 & 0 & -3 \end{array} \right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1/2 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right)$$